AXIOMS FOR COMPETITIVE EQUILIBRIUM

with a perspective on increasing returns to scale

by Thomas Cool* July 11 1990

Abstract

Modern Walrasian economics investigates the consequences of the Perfect Competition Hypothesis (PCH) that agents regard prices as given, but not its *explanation*. The latter however is crucial. A point commonly directed at increasing returns to scale also applies to constant and decreasing returns: why don't people pool their property rights, and reap the benefits of a monopoly ? Hence, the returns to scale are immaterial to the argument, and we need a proper description of the behaviour of economic agents *in general*. Giving an explanation must be understood as: splitting up axioms, to reveal the deeper truth behind them. Entrepreneurial love of independence is shown to be an property of the utility functions, axiomatic for CE. Its use has three advantages: (a) didactic clarity on the case of increasing returns, (b) we can better relate to the classical economists, with their explicit discussion of the entrepreneur (Walras, Marshall), (c) there is room for progress, with a more general formulation of individual preferences about the way of cooperation; with a link to organisation theory and economic systems theory. The analysis presented remains in the realm of constant returns, but it gives a good perspective on solutions for increasing returns.

*) Thanking C. Weddepohl for valuable discourse

Contents

Introduction

Originally, Cool (1985) noted that empirical economics would welcome the combination of competitive equilibrium (CE) and increasing returns to scale (IRTS), but he also noted that many economic theorists seemed to believe that this combination was logically impossible. In fact, also in practical discussions one did encounter various arguments which seemed an impediment to proper empirical conclusions. A first insight forwarded was, that a resort to the assumption of constant returns to scale (CRTS) would be improper, since one would solve a behavioural problem by means of a technological assumption.

This present paper [1]) will continue on that line of reasoning. Under CRTS the behavioural question can be posed: why don't people pool their resources, and reap the benefits of a monopoly ?

The question originally arose in the context of a discussion on IRTS: since many people think that IRTS would lead to a monopoly. In the process of conjecture and refutation it however is natural to ask: why not in CRTS ? And then we see that the returns to scale actually are ***immaterial*** to the argument, and that we need a proper description of the behaviour of economic agents ***in general.***

It so appears that modern Walrasian economics investigates the ***consequences*** of the Perfect Competition Hypothesis (PCH) that agents regard prices as given, but not its ***explanation.***

Giving an explanation must be understood as: splitting up axioms, to reveal the deeper truth behind them. So that is what we are going to do here.

Conventional competitive equilibrium

The conventional definition of (axioms for) a competitive equilibrium CE for example can be found in Debreu (1982) p 704.

An *economy* is given by

$$E = \{ \{(X, \preceq, e)_i\} \{s_{ij}\} \{Y_j\} \}$$

where X is a consumption set, $\preceq$ a preference relation, e the endowments, s the shares, and Y the production set, for m consumers and n producers.

A *state of the economy* is given by

$$S = \{ \{x_i\} \{y_j\} \quad p \}$$

A state $S^* = \{ \{x^*_i\} \{y^*_j\} \quad p^* \}$ is an *equilibrium* when

(A) for every i: x^*_i is the best element in the budget

$$\{x \text{ in } X_i \mid p^*x \leq p^*e_i + \sum_j (s_{ij}\, p^*y^*_j)\}$$

(B) for every j: y^*_j maximizes p^*y_j on Y_j

(C) the state is feasible (supply equals demand)

$$\sum_i (x^* - e)_i = \sum_j y^*_j$$

Debreu adds: "The equilibrium defined by conditions A, B and C is competitive in the sense that every agent behaves as if he had no influence on prices and considers them given when choosing his own action." Also Takayama (1974) and Mas-Colell (1980) observe that modern Walrasian economics investigates the *consequences* of the hypothesis that agents regard prices as given, but not its *explanation.*

Some explanations

Above, obviously, p^* is postulated as given for conditions A, B and C. Debreu's additional instruction is crucial, since otherwise the problem would be misspecified. Now it is only ***partly specified.*** The correct approach would be to introduce explanations why each agent behaves 'as if' prices are given. Various authors recognize this, and by inserting additional verbal instructions they try to make up for the limitations in the mathematics department.

This critique is of a general nature, since also oligopoly theory postulates the conditions and does not explain them. But it is not sufficient to simply assume a regime. One has to clarify how economic agents ***know*** in what regime they live. [2]) Of course, such explanation might ask for a Hicksian historical explanation, but let us first try the mathematics.

Looking at the *economy* definition, we note that agents would have various instruments for the maximization of their objective function: quantities and prices, but also the selection of the trading partners. It is only in CE that prices appear (a) uniform, (b) ineffective. The proper question is: what conditions make that agents have no influence on the market price ?

In a systematic approach, we can use monopolistic competition. There are reaction functions for the instruments, and, in some limit case, goods are identical, and reactions to price deviations can be catastrophic (*all* or x^*_i or *nothing*): making it CE. The question thus can be reformulated as: what conditions make such reaction functions catastrophic ?

The distribution of the endowments across agents for example is no free matter like the general statement might suggest: for if

one agent were to have all endowments, one would hardly call the economy 'competitive'.[3] The *tatonnement* process is only instrumental in getting *uniform* prices, but there is nothing in the process which eliminates distributional effects.

IRTS would not be a problem under monopolistic competition, so, how to prevent it becoming a problem under CE ? Technical bottlenecks and installment might limit the pooling of resources; but this is technical, and not behavioural. An alternative is Weddepohl's (1974) (1990) consideration of marketing costs; but this doesn't touch upon the more fundamental possibility of pooling property rights (shares s_{ij}) and then reap monopoly profits.

Takayama (1974) p 226 makes the basic observation here: "the fundamental notion of a competitive equilibrium is that each agent is a price taker rather than that the influence of each agent is nil. The latter implies the former, but not vice versa. It is true that each agent would be silly to act as a price taker if he can influence prices; but the amount of his influence may be so small and the cost of obtaining information with regard to his influence and of forming coalitions may be so large that each agent may end up acting as a price taker. In other words, one can argue that the influence of each agent in a competitive market is nil, not because he is atomless, but because the high cost of a coalition forces him to be a price taker."

This is a key statement. An proper development of CE would formalize such key words as *influence, information, coalition, size, costs*. However, again much emphasis is put on *technological* assumptions concerning information processing and resulting costs. Much better, we would like to see *behavioural* assumptions.

Consider also, that it would always be possible to buy shares in old firms by means of shares in a new monopoly, and that there are little costs involved in this. The invisible hand however causes that when you offer to take over a firm, it comes with a counterproposal. We therefor arrive at the idea that entrepreneurial love of independence would seem to be a major behavioural force.

We are not satisfied with monopolistic competition just to allow for IRTS, for it is also technical. We would like to link to classical writers, and have a simple combination of IRTS and CE.

A possible approach to IRTS

Having reached this point in the discussion, one will be able to understand a possible approach to CE & IRTS. It holds that profits are a price too, and that *if* it is assumed in CE that prices are given, *then* it is 'out of order' to regard profits as going to infinity. [4]) In other words: *if* so many essential behavioural assumptions are not explicitly taken care of, *then* it would not be logical to create only problems with IRTS but not with other issues.

This approach of course is second-best to actually redefining CE in such a manner (a) that the older definition indeed appears to be a natural simplification, (b) that it would turn the second-best into a obvious triviality (which it yet may not really be). In the remainder of this discussion we attempt such redefinition, and try to reveal the 'true' axioms of CE. Some more verbal arguments are put in appendices A and B.

Entrepreneurial love of independence

Entrepreneurial love of independence would be a behavioural axiom, since it would be a property of the utility functions.

Of course, it might be explained itself, to some extend, by *internal* costs. So then we have *costs* again, like in Takayama's quote. When a person doesn't like to be bossed around, doesn't like the verbal haggling in monopoly board meetings (not only because he looses), and prefers the simplicity of direct exchange on the market, one might translate that as *internal* economic choice. However, this a chicken and egg problem; it is a play with words and it doesn't solve the fundamental issue. By analogy, we might point to the problem that money itself may not enter the utility function, but can be made to do so, by means of integration over a lifetime (Grandmont (1983)), resulting into liquidity preference. One has some freedom to choose a point of vantage. But that doesn't mean that there is not a most basic vantage, and the following would show that the appreciation of cooperation is a fundamental property.

On the structure of utility functions

Utility likely is lexicographic. That is, all a-things are more important than any b-thing . Regard two arguments.

Arrow (1951) [5]) draws on some historical authors to highlight the issue. On p82 he discusses Kant's distinction in technical, pragmatic and moral imperatives. In this line, Cool (1980) relates 'pragmatic' preference to 'moral' *deontic logic*. Let *Op* denote *One ought to do p* ; let *Pp* denote *It is permitted to do p* ; let ˜ denote negation; and of course *p* is a proposition. The exemplary statement is *Op <=> ˜(P˜p)* and one sees that *Pp & P˜p* expresses moral indifference. The thing to see is that one can relate these deontic concepts to economic preference. With *xRy* meaning that *x* is at least as good as *y*, one gets: *Pp <=> pR(˜p)*. In a short discussion with deontic expert Von Wright, he and I agreed that a reduction of deontic logic to preference is not in the spirit of both subjects, *vide* the Kantian argument; but that it nevertheless might apply for utility functions with a clear top *(satisfied)*, provided such top can be called the *summum bonum*. It is an argument to regard a utility *U(x,y)* as lexicographic in *x* and *y*.

The second issue is Arrow's discussion on p91: "From a logical point of view, some care has to be taken in defining the decision process since the choice of decision process in any given case is made by a decision process. There is no deep circularity here, however. If x is the vector describing a possible social state, let x_1 be the components of that vector which are not decision processes; in general, let x_n be the process of deciding among the alternative possible x_{n-1}'s. (...) In describing the United States Government, we might say that x_1 is a proposed bill

or, more precisely, the proposed bill taken into conjunction with all the legislation now on the books; x_2 is the process by which bills are enacted into law by Congress and the President; x_3 is the process of choosing a Congress and President, set down by the Constitution; and x_4 is the process of constitutional amendment." Arrow here refers to Rousseau: "The law of plurality of votes is itself established by agreement, and supposes unanimity at least in the beginning." Gilles (1990) is a recent exponent of this line of thinking about the *organisation* of economic activity.

The argument is useful for us, for we note that organisation will have an impact on utility. Firstly, it enhances the case for lexicography. Secondly, it becomes essential to note that so-called 'competition' is a way of cooperation. *Trade* is cooperation. By specializing, people take the risk of interdependence. In that sense, I do not like the word 'non-cooperative games'; and economic life would rather be called the 'specialization game'. Like primitive money (gold), such cooperation will be appreciated (or not) for its own properties. It enters the utility function.

Renewed: competitive equilibrium

An *economy* is given by

$$\underline{E} = \{ \{(X, \leq, e)_i\} \quad \{s, R\}_{ij} \quad \{Y_j\} \}$$

with R_{ij} a preference relation.

A *state of the economy* is given by

$$\underline{S} = \{ \{x, \underline{e}\}_i \quad \{y, q\}_j \quad \{s_{ij}\} \quad p \}$$

A state $\underline{S}^* = \{ \{x^*, e^*\}_i \quad \{y^*, q^*\}_j \quad \{s^*_{ij}\} \quad p^* \}$ is an *equilibrium* when

(A) for every i: x^*_i is the best element $(x^*_i \succeq_i x)$ in the budget

$$\{x \text{ in } X_i \mid p^*x \leq p^*e^*_i + \sum_j (s^*_{ij}\, p^*y^*_j)\}$$

(B) for every j: y^*_j maximizes p^*y_j on Y_j

(C) the state is feasible (supply equals demand)

$$\sum_i (x^* - e)_i = \sum_j y^*_j$$

(D) for every i: $\{s^*_{ij}\}$ is the best portfolio $(s^*_{ij} R_{ij} s_{ij})$ in:

$$\sum_j (q^*_j\, s^*_{ij}\, p^*y^*_j) = p^*(e-e^*)_i + \sum_j (q^*_j\, s_{ij}\, p^*y^*_j)$$

We might have introduced preferences over all transactions, but for our purposes it suffices to introduce both preferences over shares, and a market place to make these non-trivial. A part of initial endowments $p^*(e-e^*)_i$ is used for share purchases, but we allow for $e_i < e^*_i$ so that a share transaction profit can be used in the purchase of commodities. The reallocation of initial $\{s_{ij}\}$ creates share or dividend prices $\{q^*_j\}$. Often people are only interested in the money value, $q^*_j = 1$, so that the budget in (A) does not change. But the personal appreciation of the

portfolio may differ from its monetary value. If all, but one person, hate to be associated with a specific factory *j*, then this one person could command a very low price for the shares.

For an ***existence*** proof, trying for IRTS leads to far. Therefor, we assume the same properties (axioms) of $\underline{E}$ as Debreu *op.cit.*, then rewrite the problem into the conventional one, and then we apply the Debreu proof of existence of equilibrium. It already is obvious that for given $(s^*_{ij}\ e^*_i)$ the first part (A-C) reduces to the old problem; and the second part (D) is a reshuffling of shares and hence a similar problem. But it is better to pool items:

$$E' = (\ ((X\ I),\ (\leq R_j),\ (e\ s_j))_i)\quad (s_{ij}\ 0)\quad (Y_j\ 0)\ )$$

with $I_i = [0,1]_r$. The occurrence of s in two places is no problem, since it are ***given*** endowments. Hence we have the old problem, and thus equilibrium exists. Note that, hence, the price of s_{ij} (i.e. $q^*_j\ p^*y^*_j$) would be considered as ***given***.

Individual preferences, for both ***goods*** and ***types of shares***, thus also determine the balance between income and the various intensities of cooperation. Thus we have a proper development of (corruptible) entrepreneurial love of independence.

We cannot say anything about IRTS, since the Debreu proof uses CRTS. But we wanted to show (a) the ***given-ness*** of profits, (b) a normal development of CE theory, ***too***, might need specific properties in the utility functions, in order to give an ***explanation*** for the PCH of given prices. And indeed, since we do

not want PCH become irrational, a *hidden axiom* is revealed to be:

(E): *$[R_{ij}]$* does not result into all shares in the hands of a few.

Conclusion

We wanted a more general description of behaviour, so that perhaps we might explain the Perfect Competition Hypothesis that prices are given. We succeeded in expanding our behavioural assumptions about the pooling of property rights. Then we showed that we would need an axiom of 'entrepreneurial love of independence', to prevent the economy from collapsing into oligopoly or worse, and making the PCH irrational.

This is a partial result, since we have no result on IRTS, and, worse, we still have not derived the PCH from first principles. For this, it would be needed to let agents use both prices and quantities as their instruments, and then deduce that they only loose from using prices. As said, Nieuwenhuis (1990) is promising for this approach. But our partial result is still very useful, since it uncovers behavioural assumptions, i.e. properties of the utility functions, which would always remain axiomatic for CE.

Its use has three advantages: (a) didactic clarity on the case of increasing returns, and a perspective on solutions, (b) we can better relate to the classical economists, with their explicit discussion of the entrepreneur (Walras, Marshall), (c) there is room for progress, with a more general formulation of individual preferences about the way of cooperation; with a link to organisation theory and economic systems theory.

Appendix A

A short history of bewilderment on CE and IRTS

1. Concept

Production is essentially only the reshuffling of mass and energy. What physically goes in (in kilograms and ergs) also comes out (in kilograms and ergs), so that in this sense Constant Returns to Scale (CRTS) exist by definition. For two reasons this happy state of affairs breaks down. Firstly, as long as free disposal is possible, economists concentrate on the part that cannot be freely disposed off. Secondly, economists are interested in goods rather than their composition. Hence the economic problem is defined to be different from the physical one, and there is sufficient reason to allow for nonconstancy in the RTS. For example, there is the possibility to have fixed installment costs, which creates the issue that one could think of a definition of marginal costs which makes those come out below average costs.

2. Some differing views

It has been the intuition of many that only constant or decreasing RTS would be realistic. Debreu (1982, p 711) for example speaks about a 'law of non-IRTS'. As reported by Dooley (1985) and Maneschi (1986), writers like Marshall and Sraffa appear to have considered IRTS, but had problems of integration of IRTS with competitive equilibrium (CE); so that the 'law' derives from logical complexity rather than from intuition. Marshall would have ended up with the view that observable RTS are determined by

the age of the entrepreneur: as a youngster the entrepreneur has difficulty with getting established, in middle age he can reap the profits of IRTS, and thereafter he and his company settle down. Lancaster (1979) is more pronounced about IRTS: he takes it for self-evident, and further neglects the problem of CE. Salant (1985, p 1177) mentions that Kaldor and Hahn accept the impossibility of IRTS & CE, while Tobin rejects it.

3. R.G.D. Allen

Of interest is that historically CRTS was a problem too. In his classical textbook R.G.D. Allen (1973) not only wrote that "the case of CRTS is seen to be of doubtful use in the theory of the firm" (p 612), but he also extended his doubt to the method of analysis: "the logical assumption in the long-run (CRTS and a linear homogeneous production function) tends to a break-down of marginal analysis. The scale of operations of a firm becomes indeterminate. Marginal analysis, therefore, is one way of describing the decision-making activities of a firm - on a simplified and approximate basis" (p 619).

The modern way to solve this case, is that theorists have dropped the need for a unique solution. This however is only part of the answer.

4. Partial analysis

The traditional partial model for a share based firm which maximizes profits by means of hired capital and labour is:

(1) production function $y = f(a,k)$

(2) β-RTS $s^{\beta}\, y = f(s.a,\ s.k)$

Euler $\beta.y = a.f_a + k.f_k$

(3) budget $\pi = py - wl - rk$

(4) first order:

$$\max_{a,k} \pi \begin{cases} \partial\pi/\partial a = 0 \quad \Rightarrow \quad p.f_a = w \\ \partial\pi/\partial k = 0 \quad \Rightarrow \quad p.f_k = r \end{cases}$$

The conventional proposition is that profits are bounded iff $\beta \leq 1$. For example Jorgenson (1984, p110) writes: "Under increasing returns and competitive markets for output and all inputs, producer equilibrium is not defined by profit maximization, since no maximum of profits exists. However, in regulated industries the price of output is set by regulatory authority. Given demand for output as a function of the regulated price, the level of output is exogenous to the producing unit." G. Heal (1986) gives a similar dynamic description of the interaction of profits and aggregate demand.

Now, Allen and Jorgenson suggest that inputs have unlimited supply. Another leading principle of this paper is that endowments are given (and that they are required in production). Hence there is a maximum to production, and the idea of exploding profits is irrational. The analysis above seems to make use of only the first order conditions for a maximum. If we look at the second order conditions, then we see them satisfied only under DRTS. This means that they are irrelevant for IRTS; and hence that all resources will be used.

One way to try to prove the 'impossibility theorem' thus is a 'reductio ad adsurdum' argument: assuming given prices leads to a price that is not given (profits). But this effort must be rejected, since it does not take into account the limited endowments (which means that also technology Y is bounded). [6]

5. Monopoly

It is argued that under IRTS there exists a bonus on forming a monopoly, leading to the breakdown of competitive conditions (e.g. Nadiri (1982) p384). But then it is not noted that property rights could be pooled under *any* returns to scale.

6. Pooling

The IRTS monopoly argument might depend upon technological pooling. But there is a problem with that concept. It is the distinct intention of 'production functions' that they ***cannot*** be pooled so easily. For example, take the various production functions for industries i under Decreasing RTS: $f_i(a_i,k_i)$. For each industry it will be obvious (from the definition of DRTS) that production can be *increased* by decentralization or depooling of resources, e.g. by halving: $2\ f_i(a_i/2,k_i/2) > f_i(a_i,k_i)$, and this to infinity. Thus, it should be clear that a production function implicitly reflects some indivisables, and that the pooling of inputs would somewhat be a contradiction in terms.

7. Some evidence

Much of this analysis originated in the context of the Dutch Paper, Printing & Publishing industry, but unfortunately there is

no evidence to quote from. Some quotable micro-economic evidence gives mixed results: name only two: Nerlove (in Hebden (1983)) finds IRTS in electric power generation; Van Helden & Musyken (1982) report that the RTS in electric power generation tend to be increasing in a technical sense, but also non-IRTS economically, since managers tend to prefer smaller units since this reduces the risk of a total power breakdown.

But we would rather be interested in the RTS at the macro level. It might indeed be an empirical economic law that actual production processes in the aggregate show CRTS or DRTS, like the 'first law of Gossen' - or perhaps one wants to refer to entropy. To be called a 'law' it of course must be investigated and tested. The practice however is that one commonly presupposes non-IRTS and then tests other things. It follows that we hardly have empirical evidence on returns to scale at the macro level. This state of affairs also derives from the noted 'impossibility theorem of IRTS & CE'. Researchers balk at the idea of testing for the existence of CE, they hence drop any illusion about IRTS. [7])

8. Myself

Cool's (1986) summary ran as follows: "There seems to exist some impossibility theorem that competitive equilibrium and increasing returns to scale cannot coexist, though it is rather obscure how the proof would look like. Two possible 'proofs' are reviewed, namely unlimited production and monopoly. The first is rejected since endowments are given, and there is no reason to suppose that agents would not know that. The second is rejected since one can presume that other factors in the background prevent

monopoly, such as government regulation or entrepreneural love for independence. Modern competitive equilibrium analysis investigates the consequences of the hypothesis that agents accept prices as given, but not the reason for the hypothesis itself. Hence the returns to scale are immaterial to competitive equilibrium."

That is, in answer to Jorgenson's conjecture above, that "no maximum of profits exists", I held that profits are a price too, and that *if* it is assumed in CE that prices are given, *then* it is 'out of order' to regard profits as going to infinity; or in other words that *if* so many essential behavioural assumptions are not explicitly taken care of, *then* it would not be fair to create only problems with IRTS but not with other issues.

Of course, there was a paradox to deal with. Equilibrium can be defined as when expectations are realized, or when there are no surprises. There seemed to be two ways to look at IRTS.

The first way is to say that a rationed maximum is also a real maximum. Everybody accepts that utility reaches a bound, even though it is maximized, and a similar thought might hold for firms. As consumers have given endowments, also the value of their shares is given, and hence the values of the firms are given. The profit motive is satisfied when the value of the firm is attained. Accordingly, when the *expectation to be rationed* is satisfied, then there is equilibrium. The second way is to hold that the profit motive is a real motive, so that managers do not regard profits as given, and so that under prices (p, w, r) they are hopelessly frustrated in their demand for inputs. Traditionally here again two possibilities arise: either one holds rationing as typical for disequilibrium, or one introduces another kind of

equilibrium (e.g. Nash) but which then need no longer be CE (in its traditional formulation, which then is exposed as deficient).

The question to be answered is why one would be happy to expect to be rationed, and why one would not change some price in order to get rid of the idea of being rationed. Here the explanation for rationing however is simple: there simply are not more endowments, and it would be irrational to think that one is richer than one really is. Thus all meddling with prices has been done and one is satisfied to accept the restriction. This is the reason why one accepts prices as given, and then analysis proceeds in the normal manner.

The bewilderment with this 1986 paper is, that I never got to publishing it.

Appendix B will contain remaining useful comments from that paper.

Appendix B

On knowledge

When one starts to explain the Perfect Competition Hypothesis, it is natural to work within the framework of 'non-cooperative games'. Especially here the idea of 'knowledge' can be important (see Johansen (1982)). In the standard formulation of CE an agent only needs to know the prices and his own activity set. In reality agents know more than that. They have some knowledge of the market, how others react to prices. Managers know more about wages and the labour market than partial analysis suggests. Additional knowledge of this kind need not be too demanding on the agents, since they already must know their activity set, and must be able to work a budget identity. The point is that there is a difference between power and knowledge. We might well assume that agents have knowledge, as long as we leave them powerless. It is rather the latter that is in need for explanation.

Regard for example the purchase of investments within such a typical game. A company could consider to outbid all others, so that by accumulating investments it would soon own the whole capital stock. However, all other companies have the same possibility to outbid others and since they know this of each other nobody will actually do it, since it only raises the price they have to pay for the investment goods.

Debreu's (1982) proof for the existence of traditional CE makes use of game theory: by way of a trick, the original problem is reformulated and then solved as a Nash equilibrium. Though this is merely a trick as yet, the idea forwarded here is that the

original problem really is a game problem, so that the solution by means of game theory is more than a trick.

There is also a more serious reason. Even when agents are powerless with regards to prices, there may still be room for some other 'power'. Even if the price of investment goods is given, the producers of investment goods still have the freedom to choose whom to sell to. It is primarily that they don't want to make themselves dependent of only one costumer which prevents the breakdown of CE.

Knowledge can be said to generate a restriction. Each producer will maximize his profits subject to his purchases, with a possibility to sell those inputs which he does not use. Once we work in a dynamic framework, where production depends upon stocks from earlier purchases, the difference is clear. The formulation of CE neglects this time dimension, which raises the question whether the associated problems are neglected too. Whatever one thinks at the micro level, at least *aggregate* supply is limited, and each producer should be conscious at least of that ! Even in the static problem the knowledge of limited aggregate endowments must trickle down to the disaggregate level. This is basic in the present problem.

At least one improvement in the formulation of CE should be that profit maximisation occurs subject to endowments. In the present formulation IRTS would induce companies to think that they live in the Land of Cockaigne. In the improved model the 'rationed' maximum would be perceived as a real maximum. With total endowments given, the distribution of them over firms would be a normal exchange problem.

Given this analysis, my 1986 preference is to maintain the description 'CE' also for cases with this kind of restriction. Of course, in the conventional formulation of CE those restrictions are not explicitly entered into the production set, but conventions can be changed (at least on such small issues like this). So point (B) in the formulation of CE above must mention the endowments which the company has been able to buy; point (C) maintains aggregate feasibility; and the distribution of endowments over firms is a normal exchange problem (with the values of the firms as given).

Footnotes

1) For the record: Cool (1985) suggested that we might use an equilibrium concept 'very close' to CE. But is obvious that we win a lot in clarity and historical consistency when it appears possible to retain the notion of CE for general application, also for IRTS. Cool (1986) has the same argument as the present paper, in that returns to scale are immaterial to CE and that one needs entrepreneurial love of independence. See the Appendices. But I never got to publishing it. The present paper is better since it gives an explicit formulation of 'actual' CE.

2) See Gilles (1990), of which I've read only the introductory parts. Nieuwenhuis (1990) analysis of oligopoly seems promising of solving rather general problems, and likely those of this paper too. But our approach has some important different features.

3) One way to express 'smallness' might be the following. Let p^k be the equilibrium price when k does not participate in the economy. When $|p - p^k| < \epsilon$ then k has only a marginal effect on the economy. When k for example only affects the 80th decimal of every price, then for all practical purposes the equilibrium price is not really changed. But, where is the elegance ?

4) In terms of each production set Y, one generally defines IRTS as: if $y \in Y$ and $a>1$ then $ay \in Y$. If $py > 0$ then there would be no limit to production. Of course there will be boundaries in Y which limit actual production. The problem actually is, that expectations would be unlimited. See footnote 6.

5) Cool (1990) rejects Arrow's main proposition, but this paper shows that his book remains important.

6) However, there is a difference between actual production and expected production. Allen and Jorgenson likely are conscious of boundaries, and they actually point out that ***expected*** production would be infinite. This gives the real problem. In the model the prices of shares are given but producers perceive them to go to infinity. My problem with this state of affairs is that this latter belief actually is irrational, since actual profits will be limited. So the real problem is perhaps not the problem within the model, but rather that the model is wrong (too simple).

7) Cool (1986): "If this description is correct then theorists have some work to do. First of all there is actually some confusion whether the (impossibility) theorem is correctly proven. The state of art is best described by saying that we have a proof that CRTS and DRTS can coexist with CE, but apparently the necessity is still dubious. Secondly, if the theorem were correct, then the question arises if IRTS can coexist with some other kind of equilibrium - which might very well be the one now in real existence. Up to now theory may have made it easy for itself." I hope to have resolved this now.

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