

Marcus learns counting and arithmetic with ti in Danish

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<http://thomascool.eu>

See: *A child wants nice and no mean numbers*

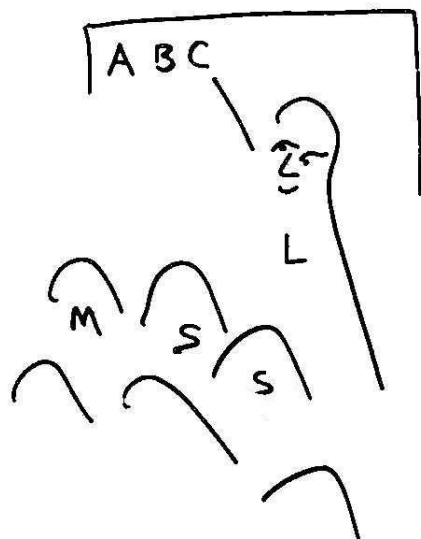
<http://thomascool.eu/Papers/NiceNumbers/Index.html>

Danish can use current *ti* as below, but also has the option to use English *ten*.

For the ordinals a suggestion would be to use *-de*.

For fractions do not use the ordinals ("a fifth"), but use $y x^H = y / x = "y \text{ per } x"$.

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1. Marcus and his friends at school

Marcus is now at school.

His friends Sam and Susan are in his class too.

They have reading, writing and arithmetic.

The teacher is called Linda.

Miss Linda shows how to do it.



2. Marcus knows ti digits

Marcus knows the letters of the alphabet.

He uses the letters to make words.

Marcus also knows the ti digits.

We use these to make the first numbers.

nul	0
en	1
to	2
tre	3
fire	4
fem	5
seks	6
syv	7
otte	8
ni	9
ti	10

Do you see the difference between a digit and a number ?

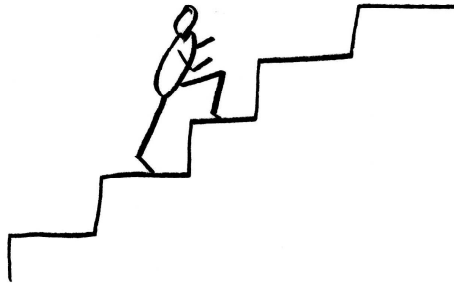
A number is recorded with the digits.

A hand has 5 fingers.

To hands have 10 fingers.

When you calculate with nul then you better use candy. (It must be able to disappear.)

It is Marcus's birthday and he brought cookies !



3. Count and add

Numbers can be used for counting.

You count when you say: 0, 1, 2, 3, 4, 5, ... and so on.

Numbers can be used for addition.

You add when you say plus and then what it adds up to.

Or when you write numbers with + and then =.

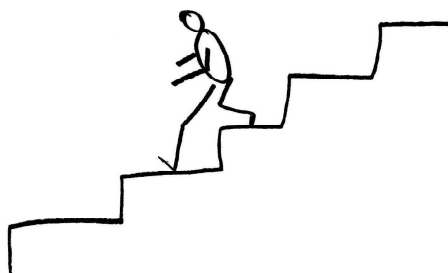
This adds 1.

nul plus en er en	$0 + 1 = 1$
en plus en er to	$1 + 1 = 2$
to plus en er tre	$2 + 1 = 3$
tre plus en er fire	$3 + 1 = 4$
fire plus en er fem	$4 + 1 = 5$
fem plus en er seks	$5 + 1 = 6$
seks plus en er syv	$6 + 1 = 7$
syv plus en er otte	$7 + 1 = 8$
otte plus en er ni	$8 + 1 = 9$
ni plus en er ti	$9 + 1 = 10$

You can add also in a column.

number	1	2	3	4	5	6	7	8	9
plus	1	1	1	1	1	1	1	1	1
is	2	3	4	5	6	7	8	9	10

You may switch the first and second rows, with the same outcome.



4. Count down and subtract

Numbers can be used to count down.

This is when you say: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0

Numbers can be used for subtraction.

You subtract when you say minus and then what is the difference.

Or when you write numbers with – and then =.

This subtracts 1.

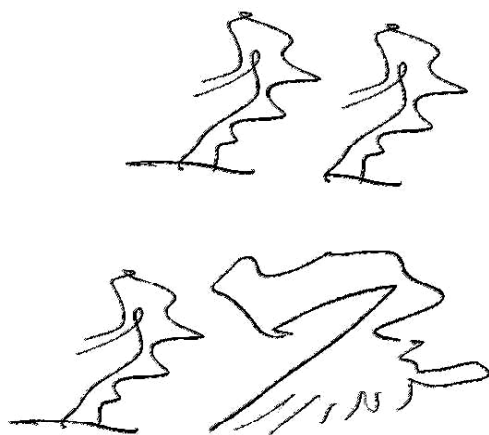
en minus en er nul	$1 - 1 = 0$
to minus en er en	$2 - 1 = 1$
tre minus en er to	$3 - 1 = 2$
fire minus en er tre	$4 - 1 = 3$
fem minus en er fire	$5 - 1 = 4$
seks minus en er fem	$6 - 1 = 5$
syv minus en er seks	$7 - 1 = 6$
otte minus en er syv	$8 - 1 = 7$
ni minus en er otte	$9 - 1 = 8$
ti minus en er ni	$10 - 1 = 9$

Check: $9 - 2 = 7$ because $7 + 2 = 9$.

You can subtract also in a column.

number	1	2	3	4	5	6	7	8	9
minus	1	1	1	1	1	1	1	1	1
is	0	1	2	3	4	5	6	7	8

You may not switch the first and second rows, because the outcome is different. (You will learn this later on.)



5. From ti to to·ti

Sam says: ti is the highest number.

Not true, Marcus says, elleve is higher.

Elleve is a weird number, Susan says.

It is the same as ti & en but people also say elleve.

Yes, Marcus says, for ti & to they say tolv.

That is easy for telling the hour.

number	10	10	10	10	10	10	10	10	10	10
plus	1	2	3	4	5	6	7	8	9	10
is	11	12	13	14	15	16	17	18	19	20

Ti plus ti is to·ti. You write a dot but don't say it.

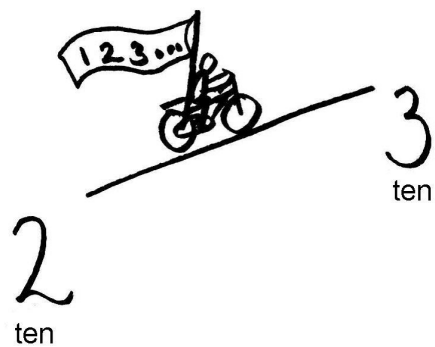
Miss Linda explains that people say the numbers in different orders. Then the names sound differently. It is useful to know this. But words like elleve and tolv will not be used in calculation.

Marcus, Sam and Susan learn the numbers to to·ti.

They also learn that they can say tyve. But not in calculation.

Reverse order but not in calculation

ti	10	ti
ti & en	11	elleve
ti & to	12	tolv
ti & tre	13	tretten
ti & fire	14	fjorten
ti & fem	15	femten
ti & seks	16	seksten
ti & syv	17	syttten
ti & otte	18	atten
ti & ni	19	nitten
to·ti	20	tyve



6. From to·ti to tre·ti

Sam says: to·ti is the highest number.

Not true, Marcus says.

To·ti plus en er to·ti & en.

This is higher.

And so on, Marcus says.

Miss Linda explains that people say *en-og-tyve* in reverse.

But not in calculation.

number	20	20	20	20	20	20	20	20	20	20
plus	1	2	3	4	5	6	7	8	9	10
is	21	22	23	24	25	26	27	28	29	30

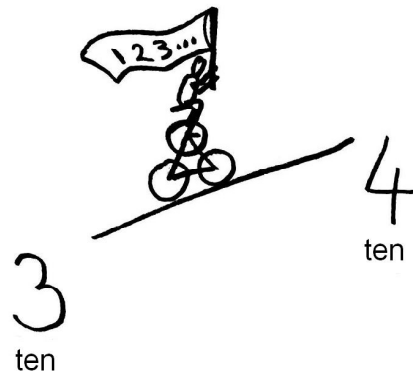
To·ti plus ti is tre·ti.

They learn that people also can say tredive. But not in calculation.

Marcus, Sam and Susan now learn the numbers to tre·ti.

Also used but not in calculation

to·ti	20	tyve
to·ti & en	21	en-og-tyve
to·ti & to	22	to-og-tyve
to·ti & tre	23	tre-og-tyve
to·ti & fire	24	fire-og-tyve
to·ti & fem	25	fem-og-tyve
to·ti & seks	26	seks-og-tyve
to·ti & syv	27	syv-og-tyve
to·ti & otte	28	otte-og-tyve
to·ti & ni	29	ni-og-tyve
tre·ti	30	tredive



7. From tre·ti to fire·ti

Sam says: tre·ti is the highest number.

Not true, Marcus says.

Tre·ti plus en er tre·ti & en.

This is higher.

And so on, Marcus says.

Sam and Susan don't believe it.

Marcus says: if you don't believe it, then calculate it yourselves.

number	30	30	30	30	30	30	30	30	30	30
plus	1	2	3	4	5	6	7	8	9	10
is	31	32	33	34	35	36	37	38	39	40

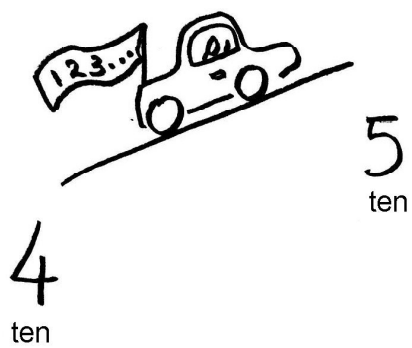
Tre·ti plus ti is fire·ti.

They learn that they also can say fyrre. But not in calculation.

Marcus, Sam and Susan now learn the numbers to fire·ti.

Also used but not in calculation

tre·ti	30	trediv
tre·ti & en	31	en-og-trediv
tre·ti & to	32	to-og-trediv
tre·ti & tre	33	tre-og-trediv
tre·ti & fire	34	fire-og-trediv
tre·ti & fem	35	fem-og-trediv
tre·ti & seks	36	seks-og-trediv
tre·ti & syv	37	syv-og-trediv
tre·ti & otte	38	otte-og-trediv
tre·ti & ni	39	ni-og-trediv
fire·ti	40	fyrre



8. From fire·ti to fem·ti

Sam says: fire·ti is the highest number.

Not true, Marcus says.

Fire·ti plus en er fire·ti & en.

And so on, Marcus says.

Sam and Susan now agree with him.

number	40	40	40	40	40	40	40	40	40	40
plus	1	2	3	4	5	6	7	8	9	10
is	41	42	43	44	45	46	47	48	49	50

Fire·ti plus ti is fem·ti.

Fem children with each ti fingers have fem·ti fingers in total.

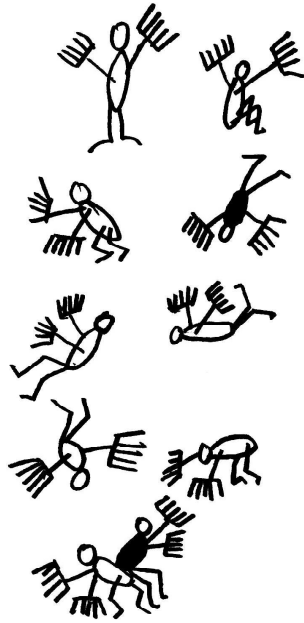
They learn to count to fem·ti.

Also used but not in calculation

fire·ti	40	fyrre
fire·ti & en	41	en-og-fyrre
fire·ti & to	42	to-og-fyrre
fire·ti & tre	43	tre-og-fyrre
fire·ti & fire	44	fire-og-fyrre
fire·ti & fem	45	fem-og-fyrre
fire·ti & seks	46	seks-og-fyrre
fire·ti & syv	47	syv-og-fyrre
fire·ti & otte	48	otte-og-fyrre
fire·ti & ni	49	ni-og-fyrre
fem·ti	50	halvtreds

Miss Linda applauds.

They are such smart kids !



9. Ti·ti is hundrede

Miss Linda says:

Shall I show you the numbers to a hundrede ?

Hundrede, Susan asks, what is that ?

Hundrede, Miss Linda explains, that is ti·ti.

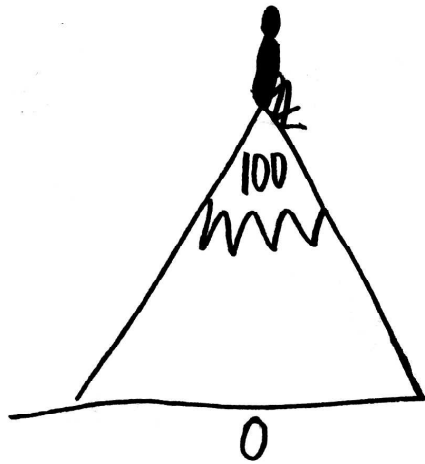
Ti children with ti fingers have ti·ti fingers jointly.

Hundrede is a word that we use in calculation too.

And so on, Marcus says, raising his hand with en finger.

Miss Linda laughs.

Yes, she says, that is a hundrede og en.



10. Hundrede og en numbers

Miss Linda shows the numbers to hundrede.

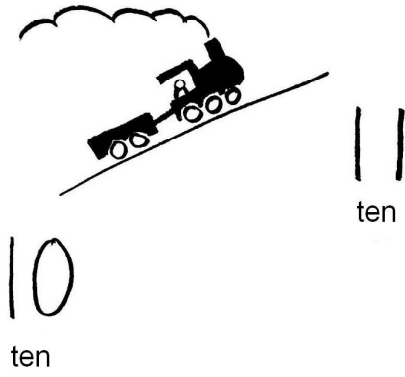
0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

These are the numbers of ti.

ti	10
to·ti	20
tre·ti	30
fire·ti	40
fem·ti	50
seks·ti	60
syv·ti	70
otte·ti	80
ni·ti	90
ti·ti, hundrede	100

Also used but not in calculation

ti
tyve
trediv
fyrre
halvtreds
tres
halvfjerds
firs
halvfems



11. Above hundrede

Sam says: hundrede is the highest.

Not true, Marcus says.

Hundrede plus en er hundert & en.

And so on, Marcus says.

Didn't you pay attention, Sam ?

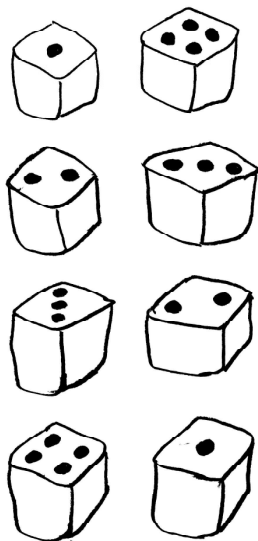
Miss Linda already said this, didn't she ?

Sam and Susan now agree with him.

Miss Linda nods. Hundert & en is 101.

number	100	100	100	100	100	100	100	100	100	100
plus	1	2	3	4	5	6	7	8	9	10
is	101	102	103	104	105	106	107	108	109	110

Miss Linda says: let us look at the numbers less than hundrede.



12. The tables of addition of ti

Miss Linda says: let us look at the table of addition.

When we add 1, 2 and 3 with themselves and each other, then we get this table.

+	1	2	3
1	$1 + 1 = 2$	$1 + 2 = 3$	$1 + 3 = 4$
2	$2 + 1 = 3$	$2 + 2 = 4$	$2 + 3 = 5$
3	$3 + 1 = 4$	$3 + 2 = 5$	$3 + 3 = 6$

And so on, Marcus says.

Miss Linda nods.

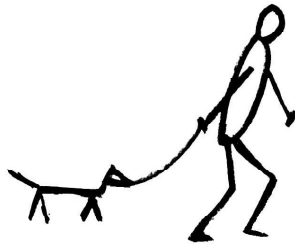
When we add the numbers to ti then we get this table.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Do you see that fem fingers plus fem fingers is ti fingers ?

And fire fingers plus seks fingers is ti fingers too.

Do you see that ti plus ti is to·ti ?



13. Mental addition in steps

Susan may pick a number. She says 4.

Sam may pick a number. He says 8.

Miss Linda ask Marcus to add these up. What is $4 + 8$?

Marcus counts down from 4 to 3.

For the second number he counts up from 8 to 9.

number	4	3
plus	8	9
<hr/>		
is		

Marcus counts down from 3 to 2, and up from 9 to 10.

number	4	3	2
plus	8	9	10
<hr/>			
is			12

Marcus looks in the table. Yes, $4 + 8 = 12$.

Miss Linda explains what is easy to do:

- If the first number is less than 5 you count down, and for the second number you count up.
- If the first number is 5 or more you count up, and for the second number you count down.



14. Mental addition with jumps

When you learn the table of addition by heart then it goes faster.

Then you don't make steps but jumps.

How do you do these sums ?

Does everyone in class have the same outcome ?

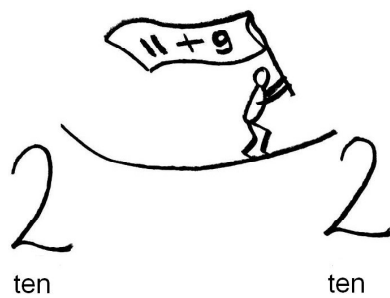
$$5 + 6 =$$

$$7 + 8 =$$

$$9 + 3 =$$

$$2 + 6 =$$

$$4 + 7 =$$



15. The table of addition of to·ti

Miss Linda says: When I use small writing then I can make the table of addition for 1 to 20.

To·ti plus to·ti is fire·ti.

+	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

Susan may pick a number. She says 9.

Sam may pick a number. He says 14.

Miss Linda asks Marcus to add these. What is $9 + 14$?

Marcus counts from 9 to 10, and down from 14 to 13.

number	9	10
plus	14	13
is	23	

Marcus checks the table. Yes, $9 + 14 = 23$.



16. Adding more numbers

Sam may pick a number. He says 7.

Susan may pick a number. She says 11.

Marcus may pick a number. He says 6. It is his seks-th birthday.

What is $7 + 11 + 6$?

The friends start adding the tre numbers.

When they find 0 or 10 then they stop changing them.

number	7	8	9	10
plus	11	10	10	10
plus	6	6	5	4
is				24

You can also add numbers one by one:

$$7 + 11 + 6 =$$

$$18 + 6 = 24$$

Another sum: $27 + 36 = \dots$?

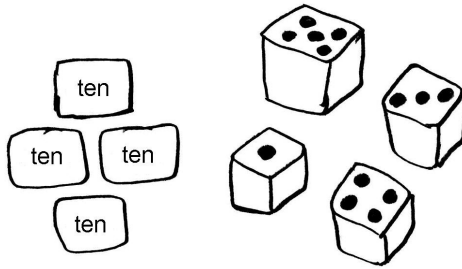
to·ti & syv plus tre·ti & seks =

step: syv plus seks = ti & tre

step: the latter ti plus to·ti plus tre·ti = seks·ti

seks·ti & tre = 63

You can do it differently but this method works always.



17. Adding many numbers

They may pick en or to numbers each.

Sam says 5 and 11. Susan says 20 and 3. Marcus says 14.

What is $5 + 11 + 20 + 3 + 14$?

The class wants to find out what these numbers add up to.

Miss Linda shows a fast way.

She takes the numbers of ti apart.

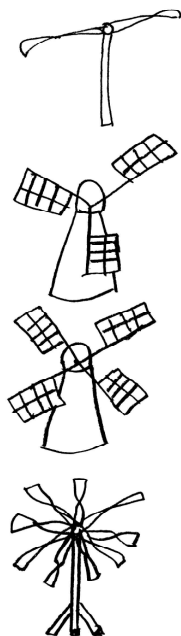
number	5		5		5
plus	11	10	1		11
plus	20	20	0		20
plus	3		3		3
plus	14	10	4		14
is		40	13		53

Fem·ti & tre. That is a high number !

Marcus shows another way to do it.

$$\begin{aligned}
 5 + 11 + 20 + 3 + 14 &= \\
 16 + 20 + 3 + 14 &= \\
 36 + 3 + 14 &= \\
 39 + 14 &= \\
 40 + 13 &= \\
 50 + 3 &= 53
 \end{aligned}$$

He thinks that the way by Miss Linda is faster.



18. Group, of, by, times

The class counts how many tiles a stoop has.

How many groups are there ? How many are there in a group ?

Here is a group of to tiles. How many tiles are there ?

En group of to = en of to = en by to = en times to = ?



en times to tiles

(en by to)

$1 \times 2 = 2$ tiles together

(en of to)

To groups of to tiles. How many tiles are there ?



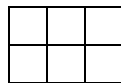
to times to tiles

(to by to)

$2 \times 2 = 4$ tiles together

(to of to)

Tre groups of to tiles. How many tiles are there ?



tre times to tiles

(tre by to)

$3 \times 2 = 6$ tiles together

(tre of to)

Fire groups of to tiles. How many tiles are there ?



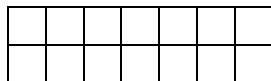
fire times to tiles

(fire by to)

$4 \times 2 = 8$ tiles together

(fire of to)

Syv groups of to tiles. How many tiles are there ?



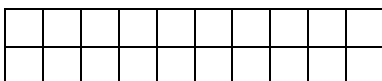
syv times to tiles

(syv by to)

$7 \times 2 = 14$ tiles together

(syv of to)

Ti groups of to tiles. How many tiles are there ?



ti times to tiles

(ti by to)

$10 \times 2 = 20$ tiles together

(ti of to)



19. Length by width

A stoop has length and width.

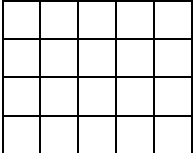
We take length horizontal (*laying*) and width vertical (*standing*).

This stoop is 5 tiles long and 4 tiles wide.

How many tiles are there ?

Wide

Long



length times width is all

5 times 4 tiles (5 by 4) (5 of 4)

$5 \times 4 = 20$ tiles all together

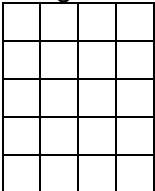
5 groups of 4 gives 20

This stoop is 4 tiles long (horizontally) and 5 tiles wide (vertically).

How many tiles are there ?

Wide

Long



length times width is all

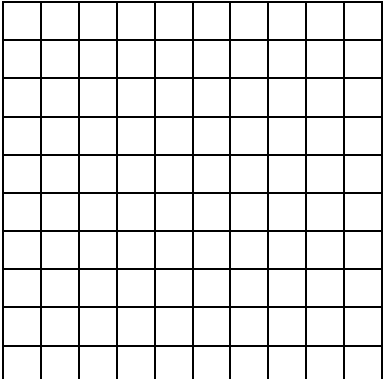
4 times 5 tiles (4 by 5) (4 of 5)

$4 \times 5 = 20$ tiles all together

4 groups of 5 gives 20

This stoop is 10 tiles long and 10 tiles wide.

How many tiles are there ?

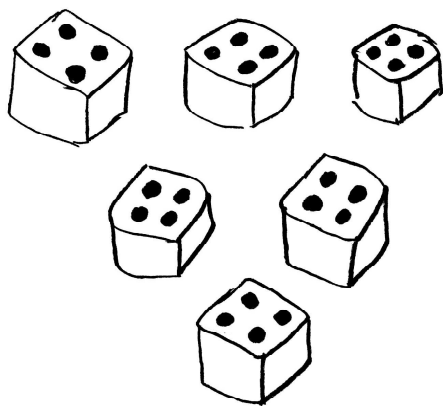


ti times ti tiles

$10 \times 10 = 100$ tiles all together

PM. What is the difference with
§10. Hundrede og en (p21) ?

Give an example when you cannot do times ?



20. The table of group, of, by, times

Miss Linda says: now we look at the table of *group, of, by, times*.

When we time 1, 2 and 3 with themselves and each other, then we get the following table.

x	1	2	3
1	$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$
2	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$
3	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$

And so on, Marcus says.

Miss Linda nods.

When we time 1 to 10 with themselves and each other, then we get the following table.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

5 Children with each 10 fingers is $5 \times 10 = 50$ fingers.

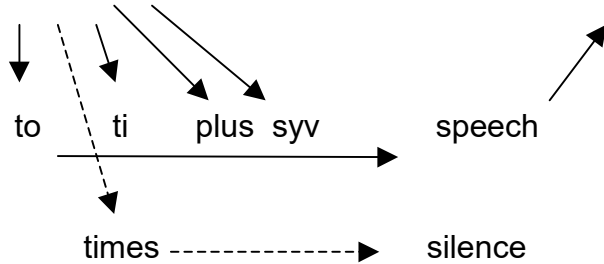
And 4 children with each 6 marbles is $4 \times 6 = 24$ marbles.



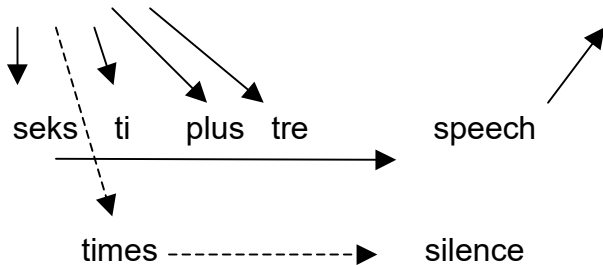
21. Speech is silver, silence is golden

Miss Linda shows these sums:

$$2 \times 10 + 7 = 20 + 7 = 27 = \text{to} \cdot \text{ti} \ \& \ \text{syv}$$



$$6 \times 10 + 3 = 60 + 3 = 63 = \text{seks} \cdot \text{ti} \ \& \ \text{tre}$$



The name of a number is how it is calculated with ti.

You can understand how numbers are spoken now that you have learned what *group*, *of*, *by*, *times* is.

Idea: write \times with red, and $+$ with green.



22. A present for Marcus

Miss Linda says:

Marcus has his birthday and I have a present for him.

Marcus, here are the very high numbers.

			<i>Short - also in calculation</i>
10^1	ti	10	ti
10^2	ti·ti	100	hundrede
10^3	ti·ti·ti	1,000	tusind
10^4	ti·ti·ti·ti	10,000	ti·tusind
10^5	ti·ti·ti·ti·ti	100,000	hundrede·tusind
10^6	ti·ti·ti·ti·ti·ti	1,000,000	million

In this way you make a high number:

number	5000	fem·tusind
plus	300	tre·hundrede
plus	80	otte·ti
plus	7	syv
<hr/>		
is	5387	fem·tusind & tre·hundrede & otte·ti & syv

Miss Linda explains:

There are almost seks·million people in Danmark.

Sam says: that is the highest number that I know.

Not true, Marcus says.

6·million plus en er 6·million & en.

And so on, Marcus says.

Miss Linda laughs.

She says: Today your name is *Marcus And so on.*



23. Marcus counts sheep

It is evening and Marcus is in bed.

His head is full of numbers.

He cannot sleep.

He counts sheep.

En, to, tre, fire, fem, ...

Tusind, tusind og en, tusind og to, ...

Million, million og en, million og to, million og tre,

Million·million, million·million og en,

Marcus says: and so on.

He falls asleep happily.