

BOYCOTT HOLLAND:
ROLE OF MATHEMATICS AND MATHEMATICS EDUCATION RESEARCH

NB. Because of developments in October 2015, a draft of this book has already been put on my website. Its contents are preliminary and only an indication.

Boycott Holland: Role of Mathematics and Mathematics Education Research

Companion to

Common Sense: Boycott Holland (2012)

&

Elegance with Substance (2009, 2015)

Mathematics and its education designed for Ladies and Gentlemen

What is wrong with mathematics education and how it can be righted

On the political economy of mathematics and its education

&

A child wants nice and no mean numbers (2015)

Thomas Colignatus

Samuel van Houten Genootschap

Colignatus is the preferred name of Thomas Cool in science. He is an econometrician (Groningen 1982) and teacher of mathematics (Leiden 2008). <http://thomascool.eu>, cool at dataweb.nl

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Prologue

In July - August 2014 I discovered that Hans Freudenthal (1905-1990) committed fraud w.r.t. mathematics education research (MER) by stealing and corrupting main ideas from Pierre van Hiele (1909-2010). My original text from July allows that Freudenthal is "no real crook"¹ but the appendix to my paper from August leaves little to guess, and new information from 2015 confirms that he was a petty crook indeed – though no demon.²

I find it absurd that the Dutch MER community wasn't shocked and didn't look deeper into the matter in 2014-2015. However, the neglect didn't surprise me.

The mathematics section at the Royal Dutch Academy of Sciences (including the humanities) (KNAW) had organised a conference on June 30 2014 on arithmetic at primary school *but none of the invited speakers was actually a teacher*. The proceedings caused me to observe a collective breach of integrity of science.³

Research mathematicians (RM) hold themselves competent and responsible for the education of mathematics, even though RM and MER are quite different things. RM is targetted at abstraction, and education is an empirical issue. Why don't RM see that their role is merely to listen and to advise ? They in 2015 have the same arrogance as Freudenthal had. See here for a scheme by Norma Presmeg.⁴

This book is essentially about *this mathematics culture of (misplaced) arrogance*. Apparently RM are aware of a hierarchy among themselves about who are the better RM, and jointly they consider themselves better than the rest of the world. Like London business bankers may look down upon commoners. Like some psychologist may think to have the better analysis and denounce others. Like some religious fanatic may think to have the better creed. All this arrogance leads nowhere, especially when it is misplaced.

This brings me to my own effort at communication with this "community".

Econometrics in this prologue, MMER in the body of the book

My degrees are for econometrics from Groningen 1982 and for teacher of mathematics from Leiden 2008. This book looks at *mathematics & mathematics education research* (MMER). This prologue allows a sketch of some circumstances w.r.t. econometrics. Had I had more success with my econometric work then this might have rubbed off on MMER. Conversely, a sorry state on MMER will increase the sorry state for economics.

After graduating in 1982, I worked at the Dutch Central Planning Bureau (CPB) in 1982-1991. In 1989-1990 I was involved with the long term scenario study 1990-2015: *Scanning the Future* and *Nederland in Drievoud* (The Netherlands in Triplo). It was a good choice back then to include the *Global Crisis scenario* for 2015.

At CPB a major aim of research is to look for consensus to get the job done. The government budget cycle requires a forecast and one doesn't have the academic luxury of questioning everything. Outside of CPB, research tends to focus on the news that deviates from consensus. A key factor for me is also what happened in 1990-1991. Developments caused me to become much more outspoken than I prefer.

¹ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

² <http://arxiv.org/abs/1408.1930> and with new data:

<https://boycottholland.wordpress.com/2015/10/20/hans-freudenthal-petty-crook-no-demon-1/>

³ <http://www.thomascool.eu/Papers/Math/2014-07-08-Colignatus-aan-KNAW-LOWI.html>

⁴ <https://boycottholland.wordpress.com/2015/10/15/pierre-van-hiele-and-annie-selden/>

In 1989, running the first projections to 2015 I observed that unemployment would remain high, which doesn't make sense, since unemployment can be solved in the medium term, especially when policy makers know about such a threat. I grew more aware of the tricky issue that wrong policy might be caused by wrong modeling. Policy makers look for economic advice but the selection of the proper economic theory and actual choice of econometric model may not be done with sufficient quality of science, and there can be political meddling. This dismal state could already be observed in the period 1970-1990. My suggestion is to extend the Trias Politica model of democracy with an Economic Supreme Court (ESC). The CPB is supervised by the Ministry of Economic Affairs but the ESC is at the level of Parliament, Executive and Judiciary, and has a scientific and thus independent role, namely guarding the quality of information for policy making.⁵

After reporting my findings in 1989, I was given a separate office with the task of *reading and writing*. It wasn't a promotion, for I wasn't allowed to run the model, and all papers that I wrote were blocked from discussion and publication, which is censorship of science. I found myself dismissed with untruths in 1991. The Judiciary assumes that the Dutch government doesn't lie. It only looks at documents put before them and doesn't test them. There is no agency with a Sherlock Holmes to investigate such abuses of power.

The CPB-director Gerrit Zalm who censored my papers and who dismissed me with the abuse of power was no scientist but made his career within the bureaucracy. CPB claims to have scientific status but Zalm neglected this, first for himself and then for me. He became minister of Finance in 1994 and currently is CEO of ABN-AMRO, not known as an establishment for scientific research.

The Dutch simply do not understand that censorship must be lifted before you can see the analysis that is being censored.

People also seem to think that an economist declares himself to be an absolute weirdo when he gives the summary that mass unemployment is a medium term problem only, and that the real problem is the structure of democratic decision making that gives not enough room for science and too much room for political manipulation.

Meeting with MMER

After CPB, I had various short term employments and projects. One possibility was to become teacher of mathematics, with a degree in Leiden 2008. For research I prefer economics over math but for teaching I prefer math over economics. Doing research in mathematics education, my expectation was that mathematical rigour and the format of *definition, theorem, proof* would enhance communication.

To my amazement mathematicians don't do the math, neglect arguments, and start abusing and slandering too.

Processes at play are:

(a) Education is an empirical issue and mathematicians are trained for abstraction. A later re-training for teacher doesn't remove their basic training. When mathematicians as teachers meet real life pupils, they resolve their cognitive dissonance by sticking to some tradition in education.

(a1) When you challenge that tradition, then you block their escape route, and you may be regarded personally as the cause for their anxiety. Punish the messenger of the bad news and it will go away.

(a2) They may simply not understand what you are speaking about when you refer to empirics. They tend to reason: If the math is okay then good students should obviously get it. If they don't get it then it is no use trying. Mathematicians

⁵ <http://thomascool.eu/Papers/Drgtpe/Index.html>.

are not aware of the empirical reality that the tradition in mathematics education is actually rather crooked so that students are right in "not getting it". A standard example: $2 + \frac{1}{2}$ is written as $2\frac{1}{2}$ but the latter suggests multiplication like $2\sqrt{2}$. History allowed some notation to become the tradition, but that tradition has not been based in empirical research in didactics.

(b) In Groningen, we students of econometrics had classes of mathematics jointly with students of mathematics, physics and astronomy. An econometrician is an applied and no pure research mathematician (RM).

(b1) My use of mathematics tends to be informal, leaving it to RM to make it as formal as they like. The RM however neglect this or make fun of it.⁶ Mathematical arrogance can turn into sickness when some RM think that they spot what they call a crank. They can use any method, including misrepresentation, to tear that supposed crank apart.

(b2) School mathematics (SM) is informal anyway. When you discuss the didactics of the latter, then it might be mistaken for your own level of mathematical competence.

Dutch culture and limits to tolerance

Holland has the reputation of tolerance but reality shows its limits. Dutch culture is quite sensitive to social tension. Don't rock the boat. Given the turbulence at CPB, people may grow annoyed by turbulence about MMER. All too soon, Dutch people may presume that there is something about my person that causes all that turbulence without useful results. Perhaps Chinese culture allows for more personal freedom.

There is no solution but for Dutch culture to grow up.

I already advised to boycotting Holland till the censorship by the directorate of the CPB is lifted. This is the main message from my weblog⁷ and also from an economics book from 2012.⁸

The reader can check the order of time and causality w.r.t. MMER too, notably w.r.t. Euclides, the journal of the Dutch association of teachers of mathematics (NVMW):

- I published *Elegance with Substance* (EWS) (2009) and had never met Ger Limpens, who in his review in Euclides 2010 started calling me names.⁹
- I published *Conquest of the Plane* (COTP) (2011) and had never met Jeroen Spandaw, who in his review in Euclides 2012 started calling me names. There are three reviews that call for reading COTP with an open mind, but precisely this slander got published here in the journal of fellow teachers of mathematics.¹⁰
- It is only fair that I protest against that abuse. The editors of Euclides didn't accept my protest however and block discussion of new books.¹¹ This is blatant censorship again. A better answer is to look for reviewers who don't slander.
- I asked the office of integrity of KNAW to look into the case. Their current decision is that slander must be accepted as part of the Dutch way of doing science. I have filed a protest, and hopefully they reconsider their view.¹²

⁶ *What a mathematician might wish to know about my work*, now included in FMNAI (2015).

⁷ <https://boycottholland.wordpress.com/about/>

⁸ <http://thomascool.eu/Papers/CSBH/Index.html>

⁹ <http://thomascool.eu/Papers/Math/Index.html>

¹⁰ <http://thomascool.eu/Papers/COTP/Index.html>

¹¹ <http://thomascool.eu/Papers/AardigeGetallen/2012-06-Gedoe-bij-Euclides.html>

¹² <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

General landscape

This prologue has painted the general landscape. Part of the landscape are the books mentioned above, generally with PDF online, while there is also *A child wants nice and no mean numbers* (CWNN) (2015).¹³ There are three issues that developed in 2015:

- Breach of integrity of science by some MR w.r.t. infinity and the inconsistency of ZFC axiomatics, see the book FMNAI and its website.¹⁴
- Breach of integrity by Jan van de Craats, professor emeritus of the University of Amsterdam, who has been leading the opposition to Hans Freudenthal's "realistic mathematics education" (RME).¹⁵ There are actually two sects: no scientists.
- More information about the situation w.r.t. Sacha la Bastide – van Gemert.¹⁶

The reader is now ready for the *Introduction* that discusses the structure of the book.

In sum

An advice is to re-engineer mathematics education. Parliaments around the world are advised to have each their own parliamentary enquiry to investigate the issue and make funds available for change. Each State may have its own State Institute for the management of mathematics education and its research, with only a limited role for mathematicians, and a major role for teachers with adequate training in empirical research.

In Holland there is a dirty math war, with a collective breach of integrity in general, and particular breaches by individuals, with slander, abuse of power, and censorship. One of the reasons is the local Freudenthal Institute with its locked-in network and Freudenthal's bossy tradition. Another reason is that if there is Dutch tolerance then it cannot deal well with the (mean spirited) arrogance of mathematicians and their maltreatment of empirical education. A third reason is that Holland may actually not be so tolerant after all.

Given the dismal situation of research integrity in Holland and the fraudster Hans Freudenthal Prize at IMU / ICMI it is advisable for the world to boycott Holland *till Holland has repaired its system of math education* (with a Simon Stevin Institute like indicated above) so that this source for pollution is redressed.

Within the environment of such a Simon Stevin Institute I will be happy to present and discuss my suggestions for improvement of mathematics education. In that case there can be unbiased empirical testing and evaluation of what works and what doesn't work.

Non-Dutch institutions like the AMS, MAA and NCTM in the USA and the ACME and ATM in the UK better do not wait for Holland but take their own responsibility to fully disclose the fraud by Freudenthal. Relevant documents must be translated by independent translators from Dutch to English. In follow-up: "scientific publications" from the Freudenthal Head in the Clouds Realistic Mathematics Institute in Utrecht must be retracted when those have no scientific value (expressing personal opinion based upon presumed expertise).

¹³ <http://thomascool.eu/Papers/NiceNumbers/Index.html>

¹⁴ <https://boycottholland.wordpress.com/2015/07/29/fmnai-is-now-available/>

¹⁵ <https://boycottholland.wordpress.com/2015/09/19/jan-van-de-craats-tortures-kids-with-fractions/>

¹⁶ <https://boycottholland.wordpress.com/2015/10/20/hans-freudenthal-petty-crook-no-demon-1/>

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Introduction & weblog context

Structure of the book

The structure of the book follows my own path and experience w.r.t. mathematics and research in mathematics education (MMER). My experience had the steps of:

- (i) doing research in mathematics education, in *business as usual*,
- (ii) discovering breaches in integrity in this field (too),
- (iii) wondering about what the situation might be internationally,
- (iv) deciding that the observed cases in Holland require documentation.

These steps make up the Parts of the book. A key role is for my weblog. Entries from there are copied to this book, and polished up, replacing links by footnotes. The articles may have a somewhat different chronology.

In retrospect I can better place the issue w.r.t. *A Logic of Exceptions* (ALOE) dating from 1980-81, when I was still a student of econometrics before working at the CPB. There is also a special position w.r.t. *Voting Theory for Democracy* (VTFD), originally developed at the CPB. I had the hope to see confirmation of this analysis by mathematicians, which would have corroborated that the CPB directorate was in error. Alas, none of that.¹⁷

Let us begin with the starting page of the weblog that explains the intention of the weblog.

Starting with: Boycott Holland

2012-02-07¹⁸

Given the developments in world democracy and the world economy it seems sensible to somewhat join the blogosphere, though I don't feel much like a blogger.

The main messages of this website are:

1. Democracy and economy are improved by upgrading the Trias Politica into a Tessera Politica, with the adoption of a constitutional Economic Supreme Court, for each democratic nation separately. See my book DRGTPE.
2. There is a new synthesis of economic theory, with various innovations, see DRGTPE again. This allows the solution of the world economic crisis and the unemployment since the mid 1970s. Clearly economic policy depends upon the theory that economic advisors adopt. If they don't have the proper theory then their advice might make matters worse.
3. The analysis on this synthesis and the Economic Supreme Court has been hit by censorship by the directorate of the Dutch Central Planning Bureau (CPB) since 1989/90. The CPB plays a key role in the preparation of Dutch economic policy. Apparently Dutch society cannot resolve this censorship of science. Hence the best advice to the world is to start boycotting Holland till that censorship is lifted. See this paper.¹⁹

¹⁷ <http://thomascool.eu/Thomas/Nederlands/Wetenschap/Artikelen/2013-02-14-PasOpMetWiskundeOverVerkiezingen.html>

¹⁸ <https://boycottholland.wordpress.com/2012/02/07/starting-with-boycott-holland/>

¹⁹ <http://thomascool.eu/Papers/Drgtpe/Crisis-2007plus/2012-01-03-Boycott-Holland.html>

4. Also my analysis on Kenneth Arrow's Impossibility Theorem was censored in 1990. See my book VTFD.
5. Supplementary is this: Part of the problem appears to lie with the attitude of mathematicians towards issues in reality. Kenneth Arrow was a mathematician who confused voting and deciding. Many economists neglect censorship of science because that doesn't feature in their mathematical models (see DRGTPE however for a model that does). Financial mathematicians helped create the world crisis. This abstract attitude of mathematicians also creates havoc in education when they meet real life pupils and students. See my books EWS and COTP, and the book ALOE on logic.

I will write on other stuff as well but the main message is to boycott Holland till the censorship at CPB is lifted and I am a free person and free scientist again. This lack of freedom also shows in that I felt forced to name this blog "boycottholland.wordpress.com" instead of a bit more positive "visitholland.wordpress.com" or something more creative. Anyway, enjoy, and start boycotting !

Mathematics and the world crisis

2012-02-08 ²⁰

It appears that mathematicians are trained for abstraction but in class they are confronted with real life pupils and students. Their tradition is to resolve their cognitive dissonance by relying on tradition. Mathematics then becomes a tool for authority and stagnation. I rather see it as an exercise in freedom, where nothing can force you to accept a proof but the proof itself.

Education in mathematics

An econometrician dismissed from a Central Planning Bureau can become a teacher of mathematics. The didactics of mathematics became a new problem area and resulted in two books *Elegance with Substance* (EWS) (2009) and *Conquest of the Plane* (COTP) (2011c) and this memo *Neoclassical mathematics for the schools* (2011d). ²¹ (The latter is superseded by the 2nd edition of EWS (2015) or FMNAI (2015).)

There are now two open minded reviews that show the strength of mathematical thinking. Gamboa (2011) ²² reviews COTP, admits to feelings of unease, but ends with a positive "enjoy". Richard Gill (2012) ²³ reviews both EWS and COTP. It was special to me that I could find something about the notion of a derivative, as developed by Newton and Leibniz, but it is even more special that Gill spends a longer open minded discussion on it. He is no expert on the didactics of mathematics and he doesn't discuss other improvements in EWS and COTP, but it is a quality review. He was trained in Cambridge, now professor of mathematical statistics in Leiden and member of the Royal Dutch Academy of Sciences (KNAW). Earlier Gill (2008) ²⁴ reviewed my book on logic *A Logic of Exceptions* (ALOE) (1981, 2007, 2011a).

Democracy

The CPB directorate not only blocked my analysis on unemployment but all my papers, also the one on Kenneth Arrow's Impossibility Theorem. Nobel Prize winners Kenneth

²⁰ <https://boycottholland.wordpress.com/2012/02/08/mathematics-and-the-world-crisis/>

²¹ <http://thomascool.eu/Papers/Math/2011-09-06-NeoclassicalMathematics.pdf>

²² <http://www.euro-math-soc.eu/node/2081>

²³ <http://www.math.leidenuniv.nl/~gill/reviewCOTP.html> has some improvements over published <http://www.nieuwarchief.nl/serie5/pdf/naw5-2012-13-1-064.pdf>

²⁴ <http://www.nieuwarchief.nl/serie5/pdf/naw5-2008-09-3-217.pdf>

Arrow and Amartya Sen misstate their results, where their interpretation does not cover the mathematics. I had hoped on some support from mathematicians. On the contrary, it appeared that social choice theorists and mathematicians did not understand the issue themselves. I developed my analysis into the book *Voting Theory for Democracy* (2001, 2007, 2011b, 2014). In 2001 I had to write this report and note of protest.²⁵ A recent event causes this short paper, Colignatus (2011e).²⁶ (The issue is handled in the 4th edition of 2014.) Our modern democracies are not so democratic while improvement is blocked by its very scholars who are not behaving so scholarly.

Finance

We need to distinguish between mathematicians and engineers, where the latter will have more eye for reality. However, 'financial engineering' may still lack the code of honour that bridge builders have, see Steinsaltz (2011), who favourably refers to Nicolas Bouleau. A good exception is also financial engineer Paul Embrechts who participated in Danielsson et al. (2001) in a warning on Basel II. On balance however, I maintain (also in *Elegance with Substance*) that part of the responsibility for the current crisis falls to "mathematicians" as well. Let them work hard towards improvement.

Economics

The world crisis has caused some economists to worry about the influence of mathematics on economic theory and practice, see this page²⁷ by professor Geoffrey Hodgson et alii. According to my analysis they do not worry enough. I am strongly in favour of mathematics, like Leibniz would say "*Let us sit down and look at the formulas*" (no quote). Only through mathematics we can establish what would be an improvement in education, democracy, finance and economics. But the latter are also empirical sciences and it would be the fallacies of miscomposition and misplaced concreteness to mistake the one for the other.

Not included

The following weblog texts have not been included but touch issues in mathematics and its philosophy and education. See e.g. the SMOJ 2012 website for what it is about. Some weblog texts are included in FMNAI 2015.

2012-02-16 Gowers and the boycott of Elsevier
 2012-12-22 The book "The simple mathematics of Jesus" (SMOJ)
 2013-02-18 Wikipedia acrobatics
 2013-10-11 A Magritte view on democracy
 2014-06-09 (In-) validating the 'Who is Bigger' algorithm
 2014-07-27 Three cheers for Holland
 2015-01-28 More praise for mathematics
 2015-04-24 The Dutch science agenda for 2015-2025: horror or opportunity ?
 2015--5-23 Abstraction vs Eugene Wigner & Edward Frenkel
 2015-05-26 Abstraction & numerical succession versus 'mathematical induction'
 2015-06-19 ZFC is inconsistent

²⁵ <http://thomascool.eu/Thomas/English/Science/Letters/SCT-working-group.html>

²⁶ <http://thomascool.eu/Papers/VTFD/2011-11-25-ShortResponse.pdf>

²⁷ <http://www.feed-charity.org/revitalizing-economics-after-the-crash.htm>

Part 1. Business as usual

Overview

Prologue from *Elegance with Substance* (2009, 2015)

Our pupils and students are best treated as ladies and gentlemen with elegance and substance. Providing them with equal mathematics is our much valued objective.

Ideally mathematics would be perfect and unchanging and just be there to be discovered. Mathematics however is as much art as discovery. It is made. It is a creation, in the way that cavemen carved their scores in bones and that we create virtual reality with supercomputers. In the interaction between what we do and what we understand almost all of the weight is on what we do, which then imprints on our mind. It appears tedious and hard work to go a bit in the reverse direction, to even get where we are now, let alone develop a notion of perfection.

Given this fragile and historic nature of mathematics it should not come as a surprise that what we currently call mathematics actually appears, on close inspection, to be often cumbersome or even outright irrational. Clarity and understanding are frequently blocked by contradictions and nonsense that are internal to current mathematics itself. Who has a problem mastering mathematics should not be surprised.

Over the years, while teaching mathematics and writing my notes that now result in these pages, there were many moments that I felt frustrated and at times even quite annoyed about the straightjacket of current mathematical conventions. One is supposed to teach mathematics but it is precisely the textbook that blocks this prospect. For many pupils and students the goal is impossible from the outset not because of their lack of capability but because of awkward conventions that only came about for historical reasons.

The flip side is that this is a Garden of Eden for didactic development. What is awkward can be hammered into something elegant. What is irrational can be turned rational and consistent. What is dark and nonsensical can be thrown out and replaced by clarity. There is beauty and satisfaction in redesign.

This didactic reconsideration also changes what we call 'mathematics'. The interaction between what we do and what we understand shifts to a new equilibrium, a higher optimum at a more agreeable level for both students and teacher. It will still be mathematics since it can be recognized as mathematics. It will be stronger and more efficient mathematics too but it will no longer be the old one.

The criterion for change lies in elegance with substance. Elegance without substance creates a dandy. Elegance ought to signal substance. Mathematics concentrates on the elegance and specific fields of study like economics concentrate on the substance. But mathematics needs to have some substance of itself too. The criterion is tricky since some people see it in the present mathematical conventions too, where awkwardness A plus awkwardness B gives inconsistency C . However, we will compare the old ways with the suggestions of the new ways and let the criterion speak for itself. This should open some eyes. Otherwise we just stay in the Garden of Eden.

Which leaves me to thank my own teachers and colleagues who trained and helped me in the old ways. A redesign starts from something and when the old is replaced then this implies that it was valuable to start with. I thank in particular my pupils and students for what they taught me.

English as a dialect of mathematics

2012-04-01 ²⁸

The West writes and reads text from the left to the right while Indian-Arabic numbers are from the right to the left. Thus 14 is *fourteen*. English switches order from 21, to *twenty-one*, while Dutch still has *een en twintig* and so on till 100.

There exists an alternative number system that satisfies didactic clarity so that pupils could learn arithmetic rather quickly. This uses the language of mathematics. The translation to English would be a mere matter of learning another dialect, which cannot be a burden in any way also given the small set of words and concepts. For example 59 can be *five·ten·nine* where English as a dialect has *fifty-nine*.

Perhaps the English and American reluctance to learn other languages and accept dialects is a larger bottleneck than possible doubts about the didactic advantages. The key notion thus is to regard English as a dialect indeed, and extend lessons on arithmetic with clarification of the dialect.

The issue came to my attention by Gladwell (2008:228): ²⁹

“Ask an English-speaking seven-year-old to add *thirty-seven* plus *twenty-two* in her head, and she has to convert the words to numbers (37 + 22). Only then can she do the math: 2 plus 7 is 9 and 30 plus 20 is 50, which makes 59. Ask an Asian child to add *three-tens-seven* and *two-tens-two*, and then the necessary equation is right there, embedded in the sentence. No number translation is necessary: It's *five-tens-nine*.”

There is not only the notation of 59 and the pronunciation, but also the notation of the pronunciation. Instead of *five-tens-nine* a better notation is *five·ten·nine*, thus no “tens” and thus the use of a high dot. The hyphen is unattractive since it is too similar to subtraction. The dot is not pronounced, like the hyphen or comma.

The choice derives from mental working space. Gladwell (2008:228): “(...) we store digits in a memory loop that runs for about two seconds.” English numbers are cumbersome to store. He quotes Stanislas Dehaene: ³⁰ “(...) the prize for efficacy goes to the Cantonese dialect of Chinese, whose brevity grants residents of Hong Kong a rocketing memory span of about 10 digits.” The quick fix is to use Cantonese internationally, yet this will meet with some bottlenecks.

This book ³¹ contains the longer discussion.

PM. Dehaene has also this useful quote here: ³²

“A lot of conceptual difficulties could be clarified if mathematicians and theoretical physicists paid more attention to the basic distinction between model and reality, a concept familiar to biologists.”

Mutatis mutandis for economics.

²⁸ <https://boycottholland.wordpress.com/2012/04/01/english-as-a-dialect-of-mathematics/>

²⁹ http://en.wikipedia.org/wiki/Malcolm_Gladwell

³⁰ http://en.wikipedia.org/wiki/Stanislas_Dehaene

³¹ The paper has become the book *A child wants nice and no mean numbers* (2015)

³² http://www.edge.org/3rd_culture/dehaene/dehaene_p5.html

The Medical School as a model for education

2014-07-18 ³³

In Medical School, doctors are trained while doing both research and treating patients. Theory and practice go hand in hand. We should have the same for education. Teachers should get their training while doing theory and learning to teach, without having to leave the building. When graduated, teachers might teach at plain schools, but keep in contact with their alma mater, and return on occasion for refresher updates.

Some speak about a new education crisis (e.g. in the USA). The above seems the best solution approach. It is also a model to reach all existing teachers who need retraining. Let us now look at the example of mathematics education.

Professor Hung-Hsi Wu ³⁴ of UC at Berkeley is involved in improving K12 math education since the early 1990's. He explains how hard this is, see two enlightening short articles, one in the *AMS Notices* 2011 ³⁵ and one interview in the *Mathematical Medley* 2012. ³⁶ These articles are in fact remarkably short for what he has to tell. Wu started out rather naively, he confesses, but his education on education makes for a good read. It is amazing that one can be so busy for 30 years with so little success while around you Apple and Google develop into multi-billion dollar companies.

Always follow the money, in math education too. A key lesson is that much is determined by textbook publishers. Math teachers are held on a leash by the answers books that the publishers provide, as an episode of *The Simpsons* shows when Bart hijacks his teacher's answers book. ³⁷ As a math teacher myself I tend to team up with my colleagues since some questions are such that you need the answers book to fathom what the question actually might be (and then rephrase it properly).

At one point, the publishers apparently even ask Wu whether he has an example textbook that they might use as a reference or standard that he wants to support. The situation in US math education appears to have become so bad that Wu discovers that he cannot point to any such book. Apparently he doesn't think about looking for a UK book or translating some from Germany or France or even Holland or Russia. In the interview, Wu explains that he only writes a teacher's education book now, and leaves it to the publishers to develop the derived books for students, with the different grade levels, teacher guides and answers books. One can imagine that this is a wise choice for what a single person can manage. It doesn't look like an encouraging situation for a nation of 317 million people. One can only hope that the publishers would indeed use quality judgement and would not be tempted to dumb things down to become acceptable to both teachers and students. In a world of free competition perhaps an English publisher would be willing to replace "rigour" by "rigor" and impose the A-levels also in the US of A.

In my book *Elegance with Substance* (2009, 2015) I advise the parliaments of democratic nations to investigate their national systems of education in mathematics. Reading the experience by Wu suggests that this still is a good advice, certainly for the US.

³³ <https://boycottholland.wordpress.com/2014/07/18/the-medical-school-as-a-model-for-education/>
This text has been edited here and also been included in *A child wants nice and no mean numbers*.

³⁴ <http://math.berkeley.edu/~wu/>

³⁵ <http://www.ams.org/notices/201103/rtx110300372p.pdf>

³⁶ <http://math.berkeley.edu/~wu/Interview-MM.pdf>

³⁷ <http://www.wired.com/2013/11/simpsons-math/>

About the subject of logic, professor Wu in the interview p14 suggests that training math teachers in mathematical logic would not be so useful. He thinks that they better experience logic in a hands-on manner, doing actual proofs. I disagree. My book *A Logic of Exceptions* (1981 unpublished, 2007, 2011) would be quite accessible for math teachers, shows how important a grasp of formal logic is, and supports the teaching of math in fundamental manner. The distinction between necessary and sufficient conditions, for example, can be understood from doing proofs in geometry or algebra, but is grasped even better when the formal reasons for that distinction are seen. I can imagine that you want to skip some parts of ALOE but it depends upon the reader what parts those are. Some might be less interested in history and philosophy and others might be less interested in proof theory. Overall I feel that I can defend ALOE as a good composition, with some new critical results too.

Thus, apart from what parliaments do, I move that the world can use more logic, even in elementary school.

Update 2015:

Editing the 2nd edition of *Elegance with Substance* (2015), now available, I was struck again by the empirical observation on the diversity of students and pupils. Evidence based education may never attain the sample sizes that are required for statistical testing of theories that allow for such diversity. This fits the Medical School model: there is an important role for individual observation and personal hands-on experience to deal with empirical variety. Methodology and statistics remain important, of course, but in balanced application.

It appears that professor Wu is updating some files. There is a rationale that such updates cause new file names and hence new links. A consequence is that old links break. My suggestion is to keep the old file names and links, and only insert the updated text. I have done so on my website and it works fine. Major changes can always be discussed in an appendix. Only fundamental new texts require new links.

One such update concerns professor Wu's text on fractions.³⁸ The text follows from professor Wu's objective to neatly develop the traditional approach. Reading it again, I am struck again by the cumbersomeness of that approach. Much more elegant is the suggestion by Pierre van Hiele to abolish fractions, and use the multiplicative form. See this short introduction³⁹ and the longer discussion in *A child wants nice and no mean numbers*.

³⁸ https://math.berkeley.edu/~wu/CCSS-Fractions_1.pdf (new link, as long as it lasts)

³⁹ <https://boycottholland.wordpress.com/2014/09/04/with-your-undivided-attention/>

Why do people hate mathematics ???

2014-10-16⁴⁰

In 2009 I wrote *Elegance with Substance* (EWS), discussing both better education in mathematics and the political economy of the mathematics industry. See the available PDF.⁴¹ Check also Steven Krantz *Through a Glass Darkly* at arXiv 2008.⁴²

The dismal state of mathematics education is generally acknowledged, essentially since Sputnik 1957. People have tried all kinds of solutions. Why do those solutions not work ?

The answer: because of barking up the wrong tree. The finding in EWS is:

1. Mathematicians are trained to think abstractly.
2. Education is an empirical issue.
3. The courses for becoming a math teacher don't *undo* what has gone wrong before.
4. When abstract thinking math teachers meet real life students, those math teachers solve their cognitive dissonance by sticking to tradition: "School Mathematics" (SM).
5. School mathematics isn't clear but collects the confusions and wreckages of math history.
6. Thus we need to re-engineer math education and reorganise the mathematics industry. One idea is that education would use the form of the Medical School: both practice and research.⁴³

EWS contains various examples where traditional math is crooked instead of clear. One example is that "two and a half" means addition and should be denoted as $2 + 1/2$, but is denoted as multiplication or "two times a half" or $2\frac{1}{2}$.

2009 + 5 = 2014

Now five years later in 2014, this explanation can be enhanced by including:

1. There is a collective failure w.r.t. the integrity of science, in that Research Mathematicians step outside of their field of expertise (RM) and make all kinds of unwarranted claims about Mathematics Education and its research (ME, MER). This aggravates the observation above that the conventional ME is lopsided to SM.
2. It is also a breach of research integrity that the warning in EWS is not responded to. When it is shown that the brakes of some kind of car don't work properly, it should be recalled – and the same for ME.
3. This especially holds in Holland. In Holland there is even explicit fraud in MER.⁴⁴
4. For the UK there is some worry, see my 2014 paper *Pierre van Hiele and David Tall: Getting the facts right*.⁴⁵
5. For the USA there is now the worry concerning professor Edward Frenkel.⁴⁶

Pierre van Hiele (1909-2010) was the greatest analyst on mathematics education of the last century, with his main thesis in 1957, coincidentally with Sputnik. However, his analysis was maltreated by Hans Freudenthal (1905-1990), who stole Van Hiele's ideas but also corrupted those – partly claiming his 'own' version but without proper reference. Van Hiele looked at the angle of *abstract versus concrete*, while Freudenthal turned this into *model versus reality*, which is didactically rather absurd, but which apparently appealed to policy makers after Sputnik 1957. Holland now has a 95% dominant

⁴⁰ <https://boycottholland.wordpress.com/2014/10/16/why-do-people-hate-mathematics-qqq/>

⁴¹ <http://thomascool.eu/Papers/Math/Index.html>

⁴² <http://arxiv.org/abs/0807.2656>

⁴³ <https://boycottholland.wordpress.com/2014/07/18/the-medical-school-as-a-model-for-education/>

⁴⁴ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

⁴⁵ <http://arxiv.org/abs/1408.1930>

⁴⁶ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

"Freudenthal Institute" that rather should be called the "Freudenthal Head in the Clouds "Realistic Mathematics" Institute" (FHCRMI). Apparently, the Dutch RM and ME / MER community is unable to resolve the issue. Internationally, IMU / ICMI (see my letter) has a "Freudenthal Medal" honoring the fraudster.⁴⁷

A leading analyst in the UK is David Tall (b. 1941)⁴⁸ who rediscovered the importance of the Van Hiele analysis, but erroneously thinks that Van Hiele was not aware of what he was doing, so that Tall claims the discovery for himself. Part of Tall's misunderstanding of the situation is the consequence of Freudenthal's abuse of Van Hiele. Professor Tall should however quickly bring out a revised 2nd edition of his 2013 book to set the record straight.

From Russia with math and confusion

I have discussed some of Frenkel's ideas. As he hasn't studied math education empirically, he is not qualified to judge, but he follows the RM arrogance to think that he is. Well, hasn't he passed through the educational system himself ? Isn't he teaching math majors now ? These are hard fallacies to crack.

Numberphile has a 9-minute interview with Frenkel, asking him: "*Why do people hate mathematics?*"⁴⁹ I leave it as an exercise to the viewer to identify the amazing number of delusions and fallacies that Frenkel mentions in this short time. Perhaps shortness invites imprecision. However, check this weblog's texts of the last week, and see that these delusions and fallacies are systematic. Just to be sure: debunking those delusions and fallacies may not be easy. If it were easy, the state of math education would not be as dismal as it is now.

To help you getting on the way, check some of these delusions or fallacies:

- The beauty of art is abused again. Math education would teach you painting fences but not the appreciation of the great results of mathematics. To some extent one can agree. Math history and some encyclopedia of math are very useful to have. But art education is not intended to get people to make masterpieces. Mathematics education is intended to help students develop their understanding and competence. These are different settings.
- Frenkel claims that everything is based upon the language of mathematics. "In a way one can survive without art. No one can survive without mathematics." Since abstraction means leaving out aspects, it should not surprise that if you start with the world and then abstract from it, then your results may indeed be relevant for 'everything'. But you cannot infer from such an abstract position that people should love their math education.
- He again is in denial of the role of mathematics in causing the economic crisis.
- The problem is often stated in the terms of 'people hate mathematics' in a manner that is not linked to mathematics education. As if there are two kinds of people, mathematicians and other – the elite versus the peasants. But the true problem is mathematics education. Math teachers have their students for some 12 years as their captive audience, and manage to turn human innate interest into said hate. By stating the problem in terms of some vague 'general audience' it becomes easier to run away from the responsibility staring you in the face, and the destruction of human lives going on in the classrooms around the world.

⁴⁷ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

⁴⁸ http://en.wikipedia.org/wiki/David_Tall

⁴⁹ <https://www.youtube.com/watch?v=Yexc19j3TjE>

Taking a blame without any consequence

There is no doubt that Frenkel respects education – though it is from personal experience and without empirical research of a national curriculum:

“Now that I’ve had students of my own, I appreciate even more what (... my teachers have ...) done for me. It’s hard work being a teacher! I guess in many ways it’s like having children. You have to sacrifice a lot, not asking for anything in return. Of course, the rewards can also be tremendous. But how do you decide in which direction to point students, when to give them a helping hand and when to throw them in deep waters and let them learn to swim on their own? This is art. No one can teach you how to do this.” (“Love & Math” p129)

The major point is this: Asked who is to blame for the dismal appreciation for mathematics (minute five) he offers himself as the scape-goat:

“If I really were to assign the blame, ... I would assign the blame to myself. And my colleagues, professional mathematicians. We don’t do nearly enough, in exposing these ideas to the public.”

Okay, so, Frenkel takes the blame. But there is no consequence. No reduction in salary. No prison term – with use of the library to start studying mathematics education. Just the burden to go out into the public and become a media star by comparing mathematics to Van Gogh, Picasso, and what other artist that can be abused and intimidated into an admiration for mathematics that they don’t understand but generally hate.

In minute six he says that the math teachers are not to blame. “They are overworked and underpaid” and “products of the same flawed system”. Thus, the idea that grown-ups should take responsibility for what they are doing, and that professional educators have an ethic to live up to, is flushed down the drain. Jesus absolves the sins of those who believe in him. The topic of discussion is reduced to “beauty”. This will generally concern topics that require an advanced university degree to understand – and that conventionally are presented in a confused manner to the general public (see yesterday).⁵⁰

About the improvement of education, Numberphile properly asks (minute seven-and-a-half): “Why has that not happened ? It seems so obvious. What you said is not like a huge conceptual link. Why isn’t it not already happened ?”

Since he has no clue about empirical science, the world turns into a conspiracy:

“Sometimes I am wondering myself why it hasn’t already happened. It is almost like a conspiracy. I mean, honestly. It is almost like there is this system of mirrors that has been created which distorts reality, that does not allow people to see what is out there.”

His closing statement turns failure on scientific integrity, fraud and dismal negligence into “irony”:

“This is the coolest stuff in the world. And yet everyone hates it. Isn’t it ironic ?”

(picture: Left: “Crucifixion” on a hypercube, Salvador Dali. Right: Edward Frenkel teaching (Source: wikipedia commons, Dali, Eget værk, Søren Fuglede Jørgensen))

PM. The link of Jesus to a scape-goat is no coincidence. December 25 falls in the sign of Capricorn and Jesus was sacrificed as the Lamb of God. See “*The simple mathematics of Jesus*” (SMOJ) for a discussion that the Bible is an astrological book – and, if you didn’t know, that astrology isn’t science.⁵¹

⁵⁰ <https://boycottholland.wordpress.com/2014/10/15/the-math-industry-of-confusing-people/>

⁵¹ <http://thomascool.eu/Papers/SMOJ/Index.html>

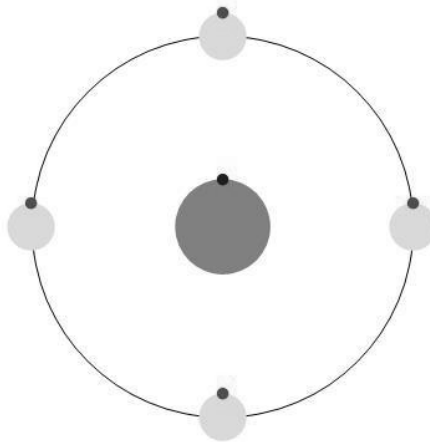
Circle and Archi measure

Einstein versus Pythagoras

2014-10-12 ⁵²

The following diagram conveys the general notion of relativity. This is not Einstein's relativity due to the constant speed of light, but it is useful to convey the notion of relativity in general.

The diagram gives an imaginary case of the Moon circling the Earth such that Earth and Moon do not rotate themselves but are always in the same orientation to the same distant stars. The Blue and Red dots are observers who remain oriented to those distant stars. An observer on Earth at the Blue dot would be able to see all sides of the Moon – assuming that Earth were transparent. For observer Blue the Moon is rotating, even though it isn't with respect to the distant stars. If the Moon is on the left hand side, Blue will see its right side. If the Moon is on the right hand side then Blue will see its left side. Similarly for top and bottom. Thus, what actually is fixed is observed as rotating. Or, if the Moon were actually rotating and Blue not, subtract one seeming rotation to eliminate this observation effect.



Moon circling the Earth while none are rotating themselves

The principle of relativity may also be explained by comparing a car driving past a house. For the observer in the house, the car has a speed. However, seen from the position of the driver, it is the house that passes by at that speed. This example also conveys that observation is relative to the position of the observer. In this case, however, the example is not that strong. The car has a brake, and the house hasn't. Thus in this case it makes more sense to say that it is the car that is causing the speed difference.

A person who turns his head sees the universe spinning around him or her. It doesn't make much sense to hold that everything is relative and that the universe is spinning around with close to infinite speed and energy. Though it would be difficult to locate, the center of the universe is a more logical point to describe events from.

⁵² <https://boycottholland.wordpress.com/2014/10/12/einstein-versus-pythagoras/>

Above diagram doesn't have the complications of a car brake or the turning of your head or Einstein's use of the constant speed of light. It shows observational relativity in terms of logic. Though the Moon does not rotate itself (Red is always oriented at the same distant stars) it seems to rotate for Blue (with the same orientation).

Pythagoras and the definition of space

Let me quote from my book *Conquest of the Plane* p85:

There is the paradoxical situation that we may take great pains to prove something that from another point of view is merely a matter of definition. The Pythagorean Theorem⁵³ is commonly expressed in terms of sides a , b and c . For the circle $c = r$. Then we get:

- Pythagoras convinces us that we have to prove that $c^2 = a^2 + b^2$
- For a distance we now define that $c^2 = a^2 + b^2$

The solution to this paradox is that Euclid used other axioms than we now do for the distance. Though Pythagoras (ca. 572 – 500 BC) lived before Euclid (around 300 BC), we can say in a figure of speech: Given the Euclidean axioms Pythagoras has to prove his Theorem. Once he got the proof he could define the circle. Without the proof he might define the circle but then would have to prove that it really exists. That said, in analytic geometry it is easier to work the other way around. Starting with formulas is a fast way to get up and running. Using distance we can define parallel lines as lines that have equal distance. With distance the circle arises naturally. The notion of distance is crucial for the Euclidean plane. We surmise that Euclid relied on a notion of distance too by using the compass.

What remains in all this is our notion of Euclidean space: a notion of straightness of lines and flatness of the plane that might derive from everyday experience but that essentially is a concept of the mind, and essentially a definition.

What you should take away from this is: the definition of 'space' is Euclidean space. If you think about 'space' then this is what you think. You cannot change what already has been defined to generate your understanding.

Einstein's historical context

As observational relativity because of the constant speed of light causes measurement errors, Einstein eliminated those errors by adapting 'space'. But can you change the notion of space if it already has been defined by Pythagoras and Euclid? An elegant way to deal with systematic measurement errors doesn't change 'space'. Something else is happening here.

Let me quote from *Conquest of the Plane* p195 that describes Albert Einstein's historical context.

A key issue in the theory of science is the issue of measurement. Physics before Newton suffered huge losses in intelligence, time and energy to discussions on unobservables and metaphysics. This in fact lasted partly into the 19th century with discussions on the 'ether'. Their solution was to put a stop to fruitless discussion and concentrate on what can be measured. You don't know what it is, but it moves this way, at that speed, and if you hit it here, then it moves there. This technical approach worked wonders, though it still seems that some

⁵³ http://en.wikipedia.org/wiki/Pythagorean_theorem

theorists assume some 'whats' to derive their theories on the 'hows' (as Bohr's atom model).

(...) A key notion below will be that physics might 'overshoot' by concentrating on measurement and by neglecting definitions and logic.

(...) Einsteins model subsequently seems to confuse the definition of space, given by the definitions of Euclid, and empirical space as measured by the instruments of physicists.

(...) Modern physicists shy away from the possibility that space and time have independent definitions within the mathematical modelling of the world. They regard space and time as what they measure. However, they don't seem to see that they can be hopelessly confused when they measure speed in meters / second while those meters and seconds change under measurement. My impression is that it is better to accept measurement error and try to explain that error.

Education in mathematics vs physics

Please observe that I am no physicist and rely for that on what I remember from gymnasium. The above is a view from the position of the education in mathematics. The views from the education in physics may be different. There may be relativity in education as well.

The above concerns a minor comment in COTP. Its real contribution lies elsewhere. PM. COTP also allows the earlier discussion of derivatives, so that physics education can start using those much earlier too.

The issue might be resolved empirically. A physicist would have to show that it is impossible to describe the measurement error in Euclidean space, so that the use of Riemann curvature is not just a historically understandable way of modelling but also necessary. It would be more interesting of course to see that the Riemann form generated other confusions.

Edward Frenkel ⁵⁴ holds that the Pythagorean Theorem meant the same to people 2500 years ago as it means to people nowadays. This doesn't seem true to fact, though of course is hard to prove. At least the above shows that we have added shades of understanding that were lacking in the past. Some historians hold that Euclid did not present a cosmology or theory of space but a theory of measuring. However, it seems that the latter presupposed the first, see point (v) here. ⁵⁵ Also, Frenkel emphasizes the importance of the Riemann model, and thus should admit that modern physicists claim another view of space than Pythagoras and Euclid, so that he cannot uphold that 'sameness'. Overall, Frenkel is a research mathematician and has no background in the empirical science of education, so he is producing a lot of nonsense. More on that later on.

⁵⁴ <http://math.berkeley.edu/~frenkel/>

⁵⁵ <https://boycottholland.wordpress.com/2014/06/29/euclids-fifth-postulate/>

Mathematical constant Archimedes = $\Theta = 2 \pi = 6.2831853...$

2012-02-18 ⁵⁶

My book *Conquest of the Plane* (COTP) uses $\Theta = 2 \pi = 6.2831853...$. My proposal in supplement to COTP is to use the name "Archimedes" for this particular symbol ("capital theta" with such assigned value). It will be a new mathematical constant. Addendum: Pronunciation "Archi" is better than full "Archimedes".

One Archi thus is the circumference of a circle with radius 1. Another relevant format is $1 / \Theta = 0.15915494...$ When you take a circle with a radius of about 16 cm then the circumference will be about 1 meter. A circle with radius r has circumference $C = r \Theta$ and surface $S = r^2 \Theta / 2$.

In wikipedia (today 2012-02-17) ⁵⁷ we can read that π is already called "Archimedes' constant". However, we commonly speak about "pi" and not about "Archimedes". Thus the name is free to use as the name of Θ .

There is some momentum in the USA to use tau, thus $\tau = 2 \pi$. Bob Palais (2001) ⁵⁸ originated the idea but used an own new symbol (pi with three legs like m), Peter Harremoës and Michael Hartl convinced him to use tau, ⁵⁹ and Vi Hart has a presentation on YouTube. ⁶⁰ One argument is that tau refers to "turn" or Greek "tornos".

However, turns are counted along the unit circumference circle $C = 1$ and not along the unit (radius) circle $r = 1$. Thus this association of tau would be confusing. Also, there is not much difference in writing r or τ . This can create a lot of confusion in handwriting, doing homework or checking exams.

Independently from Bob Palais I also came up with the idea that 2π is the proper unit of account. Looking at the various symbols available on the keyboard I rejected tau because of the similarity to r , and settled for Θ since it neatly looks like a circle. I wasn't quite happy with its uninformative name Theta but we had that also with pi or meter. Vi Hart pointed out that lower case theta is often used for angles which causes the problem of "theta Theta". This disappears when we use "Archi".

The proposal is to take the plane itself as the unit of account for angles. We know how to cut up a pie in those pointy bits radiating from the center, and we can do the same with the whole plane, getting a half plane, a quarter plane, etcetera. All those pointy bits add up to 1 plane. When we make circles we can find one with a circumference of 1 by which we can measure the angles. Comparing circles, the Archi unit shows up as a proportionality factor.

We need empirical tests whether this indeed works out better for students.

Unit circumference circle = Angular circle	Unit radius circle = Unit circle
$C = 1$ $r = 1 / \Theta$ angles α, β functions Xur and Yur	$C = \Theta$ $r = 1$ arcs $\varphi = \alpha \Theta$ and $\psi = \beta \Theta$ functions Cos and Sin

See COTP page 41. Here $Xur[\alpha] = \text{Cos}[\alpha \Theta]$. Angles can be measured by arcs or possibly be identified by them. It helps to separate the notions somewhat by putting emphasis on angles on the angular circle and arcs on the unit circle.

⁵⁶ <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

⁵⁷ http://en.wikipedia.org/wiki/Mathematical_constant

⁵⁸ <http://www.math.utah.edu/~palais/pi.html>

⁵⁹ <http://www.tauday.com/>

⁶⁰ <http://www.youtube.com/watch?v=jG7vhMMXagQ>

Archimedes revisited

2012-3-06 ⁶¹

My proposal to use the name “Archi [-medes]” for $\Theta = 6.283\dots$ got a reply from Peter Harremoës from Denmark. Peter argues that engineers and artisans in Archimedes’ time found it more efficient to measure circles by their diameter d and not with the radius $r = d / 2$, so that Archimedes calculated $\pi = \Theta r / d = \Theta / 2 = 3.141\dots$. Hence the latter number is called Archimedes’ number, historically. Peter discovered that the Persian mathematician Jamshid Al-Kashi in 1424 apparently was the first to use 6.283... as a separate number. Hence Peter suggests ⁶² to use Al-Kashi’s constant τ , where he also adopts the symbol tau as do Robert Palais, Michael Hartl and Vi Hart as shown on my proposal page.

Bear with me. I have been aware of Archimedes’ historical position, see the proposal text indeed. The point is that there is only one mathematical constant. The values 6.283... and 3.141... are mere transforms of the same constant. Thus we should select only one name. Moreover, $\Theta / 2$ would be vocalized as “one half Archimedes” such that Θ is a unit of account and not just a number discovered by some person.

It may be fun to say that Isaac Newton discovered one Newton and Alessandro Volta discovered one Volt while Archimedes discovered only one half Archimedes, but that would stretch what we mean by a mathematical constant. Archimedes really was the first to determine the mathematically correct way to catch that mathematical constant. So, there is no conflict between using the Archimedes as the unit of account and accepting that 3.141... was historically seen as Archimedes’ number.

Subsequently, Archimedes’ reasoning was didactical, since he adopted the common usage in his day of the diameter. We have switched to the radius so let us switch consistently. Perhaps Al Kashi was instrumental in that switch but he was aware of Archimedes’ important discovery and I like to think that he would agree that Archimedes receives all honour.

I have really thought deeply about tau. I really don’t mind what is actually chosen as long as it works best in education. I considered tau independently from the others but rejected it because it looks too much like r . The capital theta looks nicely like a circle. The little mark in the center is not a slash like for the diameter or crosssection \emptyset . My proposal is that we research what works empirically best in education.

It might be a nice idea to put the choices up for an opinion poll. The true vote would be to use either current π or one of the alternatives for 2π . But this vote would be biased when there is a difference in opinion about what that alternative will be. A vote now cannot be decisive since it is a matter of empirical research. However, voters can have an opinion about what should be tested in that research, or have a forecast about what would work best, at least for themselves. Thus, an opinion poll can be somewhat informative.

See this page for the vote. ⁶³

⁶¹ <https://boycottholland.wordpress.com/2012/03/06/archimedes-revisited/>

⁶² <http://www.harremoes.dk/Peter/Undervis/Turnpage/Turnpage1.html>

⁶³ <http://www.easypolls.net/poll.html?p=4f5619a1c2e1b0e4901bc494>

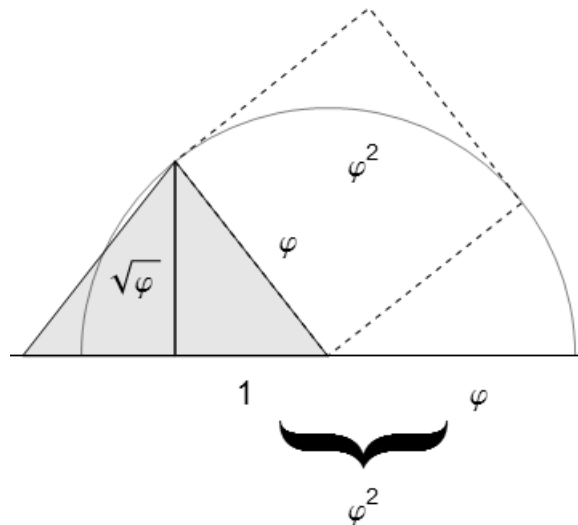
Pyramids and the meter

2013-01-27 ⁶⁴

Belgian television showed the film *The Revelation of the Pyramids*. ⁶⁵ It contains an intriguing suggestion for a mathematical relationship. Let us debunk it, though keep the intrigue.

I have three reasons to look into this. The first reason is the earlier weblog ⁶⁶ on the use of 'archi' $\Theta = 2\pi = 6.283185307\dots$ rather than π as the key mathematical concept for the measurement of the circle. Other people suggest 'tau' τ but that looks too much like the radius r and thus will cause much confusion in the classroom. The second reason is the earlier weblog on the mathematics of Jesus (SMOJ). ⁶⁷ Since the holy family fled to Egypt there is ample reason to look what was happening there. The third reason is that the film suggests that there was an ancient advanced civilisation. Since we may all be disappointed about how we ourselves are doing as a civilisation, it would be great when we could discover that others in the past have been doing much better.

We will also use 'phi' $\phi = 1.618033989\dots$ or the golden ratio. This has the property that $\phi^2 = 1 + \phi$, or alternatively that $\phi = 1 / \phi + 1$. It allows a particular interesting application of the Pythagorean Theorem. A right angled triangle with base $a = 1$ and height $b = \sqrt{\phi}$ generates a hypotenuse of $c = \sqrt{a^2 + b^2} = \sqrt{1 + \phi} = \sqrt{\phi^2} = \phi$. The associated square has the surface ϕ^2 , and by using a circle of radius ϕ we can find that same value in the length of the interval $1 + \phi$. It appears that these dimensions have been used in the pyramid of Cheops. To measure length the Egyptians used the *ell* or the (royal) *cubit* of approximately 0.5236 meters (wikipedia: between 52.3 and 52.9 cm). The pyramid of Cheops has a height of 280 cubits and a full base of 440 cubits. That shape however consists of two right angled triangles. The proper triangle has a base of 220 cubits. The ratio is $280 / 220 = 14 / 11$. It so happens that $11 * \sqrt{\phi} = 13.99221614\dots \approx 14$. Thus the Egyptians chose a ratio in integer numbers that closely matches the real value of the golden ratio.



⁶⁴ <https://boycottholland.wordpress.com/2013/01/27/pyramids-and-the-meter/>

⁶⁵ <http://vimeo.com/32828241>

⁶⁶ <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

⁶⁷ <https://boycottholland.wordpress.com/2012/12/22/the-book-the-simple-mathematics-of-jesus/>

The film *The Revelation of the Pyramids* now presents the startling equation:

$$\pi = 0.5236 + \varphi^2 \quad \text{or} \quad \pi = \text{cubit} + \varphi^2$$

Startling about this is that π and φ are pure numbers while the length of the cubit only makes sense when everything is expressed by using the meter as the standard length. The pure numbers π and φ come about as ratio's and thus by dividing lengths so that they do not depend upon any choice of measurement standard. But the value of the cubit changes if we switch from meters to feet and inches.

A first step is to check for accuracy. We find that $\pi - \varphi^2 = 0.5235586648\dots$ Thus the relation only holds by approximation, though the accuracy is eery.

A second step is to divide both sides by the cubit, or rather by the pure value $\pi - \varphi^2$. Then we find:

$$\begin{aligned} \pi / (\pi - \varphi^2) &= 1 + \varphi^2 / (\pi - \varphi^2) \\ 6.000459671\dots &= 1 + 5.000459671\dots \end{aligned}$$

There we are.

Do you see it ? Well, it took me some moments to find the proper sequence of explaining, so let us follow these steps.

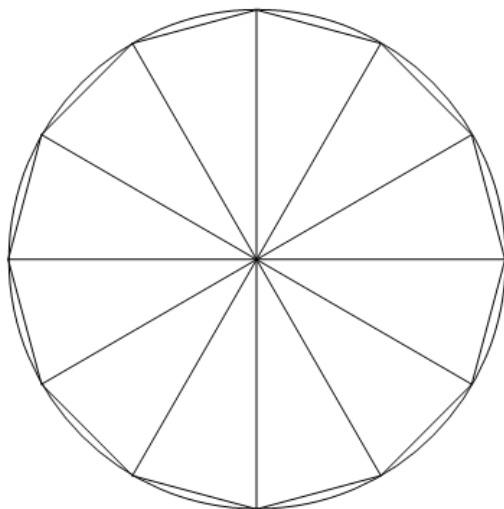
A major point is that the use of π has been playing a misleading role in this analysis. It gives only a half circle and it is much better to use Θ and the whole circle.

The first point is the surprise that $\varphi^2 / (\pi - \varphi^2) = 5.000459671\dots$ Reworked, we get:

$$\varphi^2 / \Theta \approx 5 / 12$$

What is to say about that ? Well, it apparently is a mathematical property, like $11 * \sqrt{\varphi} \approx 14$. Sometimes mathematical numbers with complex properties and long decimal expansions can get close to ratio's of specific integer values. This may be surprising, but it is a mathematical surprise. It cannot be a base for concluding that the ancient Egyptians knew about the decimal expansions of these numbers and their particular ratio. Once you decide to build a pyramid using the ratio of $14 / 11$ since it is pleasing to the eye and with structural stability, then you are stuck with the implied mathematics, but that does not imply that you know more about the implied mathematics.

Secondly, let us assume that the Egyptians had their ell or cubit as an arbitrary length (based upon the human body). They also divided the year in 12 months and day and night in 12 hours each. Thus for them it makes sense to measure the circumference of a circle by 12 cubits, like we still do in our clocks. Of these 12 pieces of a pie, six can be allocated to π , five to φ^2 , and then one remains (all with a proportionality factor).



A small problem in this discussion is that the Egyptians might use either flexible ropes (circle) or rigid yardsticks (polygon). Let us assume flexible ropes (circle) first, as they have been nicknamed 'rope-stretchers'. (See the appendix for approximation by a polygon.)

The radius r of that circle follows from $\Theta r \approx 12$ cubit, giving $r \approx 1.909859317... \text{ cubit} \approx 1.91$ cubit. For the Egyptians there was nothing special about that number for that radius. The film shows that the capstone of the pyramid would have this side. That is not inconceivable given this geometry. (If the Egyptians had $\Theta \approx 44 / 7$ from $\pi \approx 22 / 7$ then $r \approx 12 / \Theta \text{ cubit} = 12 * 7 / 44 \text{ cubit} = 21 / 11 \text{ cubit} = 1.90909 \text{ cubit}$. For them still no special value.)

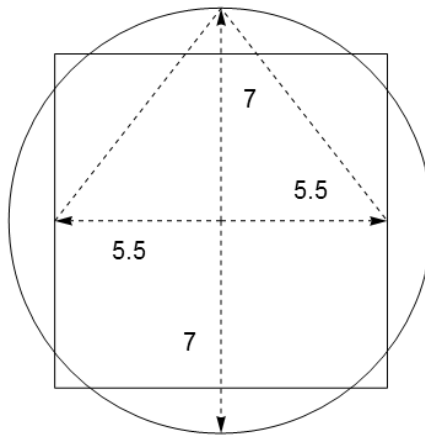
It is only for us, who have adopted the meter (rather than feet and inches), that a sense of wonder arises. For $r \approx 1.909859317... \text{ cubit} = 1.909859317... * 0.5236 = 1.000002338... \text{ meters}$! Alternatively put, if we take a circle with radius 1 meter then the division of the circumference by 12 gives us the Egyptian (royal) unit of measurement, namely via $\Theta r = 12 \text{ cubit}$ or one cubit $= \Theta / 12 = 0.5235987756....$ This uses the arcs rather than the sides of the polygon, and presumes that the flexible rope subsequently is transferred to a yardstick. (For the polygon, see the appendix.)

To understand what is happening here requires us to look into the history about the selection of the meter as the European standard of measurement. Officially, the French Academy decided in 1791 that a meter was to be one ten-millionth of the distance from the Earth's equator to the North Pole (at sea level) (wikipedia). The expedition by Napoleon to Egypt took place in 1798-1801, thus later, and the results of the new Egyptology will not have been available immediately. From this we may tend to infer that the ancient Egyptians knew about the size of the Earth and reasoned like the French. It seems more reasonable to think differently. To start with, it is already curious to take something that is difficult to measure, such as the distance from the Earth's equator to the North Pole, to define a standard. It seems more reasonable to assume that there were already circulating measures and that the story about the equator was only an embellishment. Apparently the circle with a circumference of 12 ells had been surviving over the ages and still made it into the discussion.

But the film then should be about what happened in France and not about mysteries in ancient Egypt.

NB. There is ample discussion about the measurements of the pyramid. The top is missing so we can only guess what the Egyptians intended. See the original Petrie measurements (base 9068 and height 5776 +/- 7 inches)⁶⁸ and this discussion with drawings.⁶⁹ Indeed, if the base is 220 cubits and the Egyptians had a precise estimate of $\sqrt{\phi}$ then the height would be 279.8443229 cubits, which is only a 0.06% of the whole height or one finger of a cubit short of 280. Because of this uncertainty, we cannot infer on these grounds that the Egyptians didn't have a precise estimate of ϕ . It are other documents that show us that there were severe limits to their number system. We can neither infer that they were aware of the implication that $\phi^2 = 1 + \phi$. We can observe however that they used geometry and architecture that closely matches these results. See the website by Gary Meisner for how you can create your own golden ratio paper pyramid.⁷⁰

PM. Sir Flinders Petrie (1853-1942) suggests that the basic inspiration lies in the circle rather than in the golden ratio. A circle with radius 7 has a circumference of $7 \Theta \approx 7 * 44 / 7 = 44$, using the approximation $\pi \approx 22 / 7$. This 44 gives a square with sides 11. A circle with radius 7 has the circumference of a square with sides 11. Thus we find the numbers 14 and 11 again.



The argument then is that the Great Pyramid expresses $\Theta \approx 4 * 440 / 280 = 44 / 7$, and that the golden ratio is only a by-product. If this is the case then this knowledge about Θ has been kept secret or has been lost since later documents apparently don't mention it. It is a bit curious how that knowledge can get lost when that very same pyramid is standing in front of you. Mankind however has achieved greater mysteries. Note that there is no quick transformation into $\phi^2 / \Theta \approx 5 / 12$. Via Pythagoras $\phi^2 \approx 1 + (14 / 11)^2 = 317 / 121$ and now $\phi^2 \approx 5 / 12 * 44 / 7 = 55 / 21$. For us these are approximations only but for the Egyptians it sufficed that the construction worked. The Petrie approach to start with the circle and $14 / 11$ ratio seems simplest indeed. Still, the builders will not have been insensitive to the lure of the golden ratio, and it is remarkable that they have hit upon this very shape.

Appendix

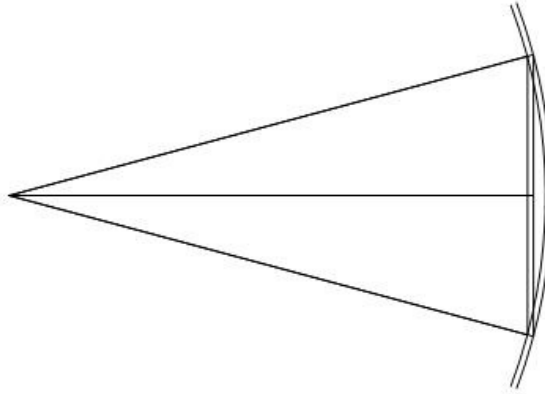
We can also assume that they did not use flexible ropes but rigid yardsticks to lay out a polygon with circumference of 12 cubits, and imagined it enclosed by a circle. We can calculate the sine of a half slice, with $\text{Sin}[\text{angle}] = h / r$. A half slice has $h = 0.5236 / 2$ and

⁶⁸ <http://www.touregypt.net/petrie/c21.htm>

⁶⁹ <http://www.khufu.dk/article/dimensions-outer.htm>

⁷⁰ <http://www.goldennumber.net/phi-pi-great-pyramid-egypt/>

the associated *angle* = $\Theta / 12 / 2$ rad or 15 degrees. We find $r = 1.01152$. The enclosing circle has a radius that is 1% or one centimeter longer than the meter.



A slice of $\Theta / 12$ of the polygon: The inner circle has $r = 1$ and $h = \sin[\Theta / 24] = 0.2588$, the outer circle has $r = 1.01152$ and $h = 0.5236 / 2 = 0.2618$.

An Archi gif, compliments to Lucas V. Barbosa

2014-07-14 ⁷¹

Given the last weblog on radians, ⁷² I noticed that Wikipedia had a nice gif animation created by Lucas V. Barbosa. ⁷³ The article even mentions: "This is a featured picture on the English language Wikipedia and is considered one of the finest images." Barbosa even made a version with tau = 2 pi. ⁷⁴ The latter is less appealing since it does not mention pi, while, of course, tau reads like radius r , and then can cause confusion (indeed, run that gif too).

It appears that Barbosa put his gif into the public domain. Thus I adapted it for Archi = Θ = 2π , including a note of reference that he did most of the creative work.

(animation: https://boycottholland.files.wordpress.com/2014/07/circle_radians_archi1.gif)

A radian is an angle measured by an arc of a circle with the same length as the radius of that circle. A full circle corresponds to an angle of 1 Archi = 2π radians. Use 1 Turn $\Leftrightarrow \Theta$ radians, so 1 radian $\Leftrightarrow 1 / \Theta$ Turn $\approx 16\%$ Turn.

Interestingly, Barbosa's original gif has a small shaded disc in the center. If we take the radius of the larger circle as $r = 1$ then we get the smaller Angular Circle ⁷⁵ in the center with $r = 1 / \Theta$ and circumference 1. My proposal is to speak about "angles" on the Angular Circle (use α and β), and to use "arc" for the radians on the Unit Circle (use ϕ and ψ). Of course, angles as measured on the Angular Circle are arcs too, but it helps being able to say that angles add up to 1 Turn and Unit Circle arcs to 1 Archi rad.

PM. The Wikipedia article ⁷⁶ I referred to has a wrong statement on dimensions (today, July 2014). For a discussion of this, see the earlier weblog entry on radians. ⁷⁷

⁷¹ <https://boycottholland.wordpress.com/2014/07/14/an-archi-gif-compliments-to-lucas-v-barbosa/>

⁷² <https://boycottholland.wordpress.com/2014/07/07/why-are-radians-not-more-natural-than-any-other-angle-unit-q/>

⁷³ http://commons.wikimedia.org/wiki/File:Circle_radians.gif

⁷⁴ http://commons.wikimedia.org/wiki/File:Circle_radians_tau.gif

⁷⁵ <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

⁷⁶ <http://en.wikipedia.org/wiki/Radian>

⁷⁷ <https://boycottholland.wordpress.com/2014/07/07/why-are-radians-not-more-natural-than-any-other-angle-unit-q/>

Why are radians not more natural than any other angle unit ?

2014-07-07⁷⁸

Blogger Zendmailer 2012 deserves huge compliments for also thinking about a circle with circumference 1, that I baptised the Angular Circle.⁷⁹ See also the figure with both the Angular Circle and the Unit Circle (radius 1) on page 36 of *Conquest of the Plane* (COTP, 2011).

Zendmailer ponders the question “*Why are radians more natural than any other angle unit?*”⁸⁰ In his words:

“I’m convinced that radians are, at the very least, the most convenient unit for angles in mathematics and physics. In addition to this I *suspect* that they are the most fundamentally natural unit for angles. What I want to know is why this is so (or why not). (...) Why not define 1 Angle as a full turn, then measure angles as a fraction of this full turn (in a similar way to measuring velocities as a fraction of the speed of light ($c = 1$)). Sure, you would have messy factors of 2π in calculus but what’s wrong with this mathematically? I think part of what I’m looking for is an explanation why the radius is the most important part of a circle.” (Physics Stack Exchange, August 6 2012)

The main thing wrong with this is that “angle” already has been defined, so that it cannot be taken as a unit of measurement. It would have been better when he had chosen 1 Turn as the unit. It is not really very wrong because if he had focussed on this longer he might well have corrected it. It is a pity that he uses 2π instead of $\Theta = 2\pi$, the unit that I call “Archi” (after Archimedes). (Others want to use tau (τ) for this, see the *American Scientist*,⁸¹ but this looks too much like r for the radius.)

By chance, if that exists, I applied the Angular Circle recently on Euclid’s fifth postulate.⁸² Check the idea in action. It is great to see that more people come up with the same kind of questions and solutions.

It is also great to see that there is room for debate. Zendmailer is convinced that radians are most convenient but there is no need for this conviction. My suggestion is to keep both circles and see which is handier on occasion. For teaching, I would start with the Angular Circle, since it would seem to be easier to calculate in 1 than in Θ . This, of course, needs testing for evidence based education.

Zendmailer rightly refers to sine and cosine functions. If we use radians, the derivative of the sine is the cosine function, so that the slope of the sine at 0 equals 1. When we use dynamic division (I refer to COTP again) then we can write $\text{Sin}[\varphi] // \varphi = 1$ at $\varphi = 0$, for φ measured in radians, using the Unit Circle. I already knew this, but Bob Palais alerted me to the phenomenon that many graphs do not show the proper slope 1 at 0.

These points arise:

1. Radians are often called dimensionless, since they arise from dividing arc by radius, thus length / length, but the arc is two-dimensional with the aspect of a turn, whence the dimension is Turn. (Addition July 29 2014: It occurred to me that this shift in focus might also be regarded in terms of the procept-theory of

⁷⁸ <https://boycottholland.wordpress.com/2014/07/07/why-are-radians-not-more-natural-than-any-other-angle-unit-q/>

⁷⁹ <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

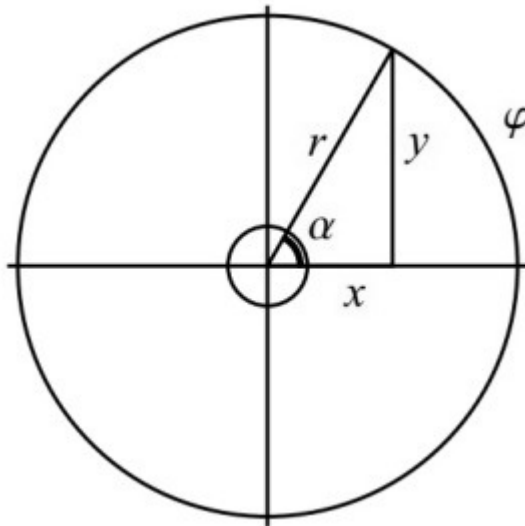
⁸⁰ <http://physics.stackexchange.com/questions/33542/why-are-radians-more-natural-than-any-other-angle-unit>

⁸¹ <http://www.scientificamerican.com/article/let-s-use-tau-it-s-easier-than-pi/>

⁸² <https://boycottholland.wordpress.com/2014/06/29/euclids-fifth-postulate/>

Gray & Tall: ⁸³ as an object we have length / length but as a process we have (length in one direction) / (length around). This may explain the difficulty for some people to 'get it'.)

2. Zendmailer uses a limit expression for $\text{Sin}[\varphi] // \varphi = 1$ at $\varphi = 0$ but skip the need for limits here.
3. Zendmailer writes $\text{Sin}[x]$ but sine and cosine represent y and x values of an angle φ .
4. For α on the Angular Circle we can find x and y values on the Unit Circle via $\text{Xur}[\alpha] = \text{Cos}[\Theta \alpha] = \text{Cos}[\varphi]$ for $\varphi = \Theta \alpha$, and similarly $\text{Yur}[\alpha] = \text{Sin}[\Theta \alpha] = \text{Sin}[\varphi]$, where the "ur" means that those x and y values are relevant for the Unit Circle. See COTP for pictures.
5. The derivatives (slopes) of Xur and Yur have a proportionality factor since these angles are measured on the Angular Circle and not on the Unit Circle. E.g. $\text{Yur}'[\alpha] = \text{Sin}'[\Theta \alpha] = \Theta \text{Cos}[\Theta \alpha] = \Theta \text{Xur}[\alpha]$.
6. Such a proportionality factor also exists for the sine of angles measured on 360 degrees. Try to figure out whether its slope at 0 is higher or lower than 1. Hint: your unit of measurement will be 1 degree.



Angular Circle ($c = 1$) and Unit Circle ($r = 1$), Conquest of the Plane p36

While trigonometry is less cluttered in using Turns and Xur and Yur , for derivatives it becomes less cluttered from using radians. Note that you can still define what the unit of measurement is, e.g. 1 cm or 1 inch, so there is no real limitation on that choice. The only limitation is the issue of consistency, that once you choose, say 1 m, then the used sine and cosine show such and such slopes.

With this established, the reading of Zendmailer's questions and the reactions should be easier.

⁸³ <http://en.wikipedia.org/wiki/Procept>

Perhaps these critical comments are still useful:

1. There is an expression “1 rad = 1”. My impression is that you should not write expressions like this, since this creates confusion. When the rad measurement is transformed from the circle arc to a straight axis in another space (where you plot the sine) then this best be indicated by a functional relationship. Subsequently, keep track of the 'turn' in the rad: $1 \text{ rad} \Leftrightarrow 1 / \Theta \text{ Turn}$. I also propose $\text{Turn} = \text{Unit (Measure / Meter) Around} = \text{UMA}$ to link up to standard measures.
2. Note Philip Oakley: “The difficulty in point 2 is that the two lengths are in independent dimensions (as in 3d space). One has just cancelled L_x/L_y and lost information for one’s dimensional analysis (this is a Physics question;-). If one did the same with Charge/Temperature it would be a gross error, but we tolerate it for length. Dimensional analysis is newer than the cubit, so the old inconsistency remains. – Philip Oakley May 11 '13 at 20:52” and “Anybody working in optics definitely cares. There are many measurements that have Angle(radians) as an integral part of their value, and it is a very common error, not spotted by dimension checking, for the angle part to be omitted, double counted, or wrongly applied. – Philip Oakley May 14 '13 at 7:30” I would like to agree but don’t know optics. Also, my impression is that L_x/L_y would cancel as straight lines though this might be different in optics; but then the better format is $2D / 1D = 1D$.
3. There is also mention of Euler’s equation, but this can also be created for Xur and Yur, and thus doesn’t carry weight for the choice between the Angular Circle and the Unit Circle.

Overall, I find that there is no “natural” choice of either Angular Circle or Unit Circle as the “natural” unit of reference. The Angular Circle seems to be best to understand how an angle is measured, the Unit Circle might reduce the clutter for who works a lot with derivatives. Dimensions however tend to arise from the field of application. Having more bodies circling a Sun at various radii destroys all simplicity anyway, especially when those appear to be no circles at all.

PM.

This Wikipedia article ⁸⁴ has a wrong statement on dimensions (today, July 2014): *“Although the radian is a unit of measure, it is a dimensionless ⁸⁵ quantity. This can be seen from the definition given earlier: the angle subtended at the centre of a circle, measured in radians, is equal to the ratio of the length of the enclosed arc to the length of the circle’s radius. Since the units of measurement cancel, this ratio is dimensionless.”* The arc is in 2D space while the radius is in 1D, and $2D / D$ still leaves a dimension. The proper dimension is Turn. Use $1 \text{ Turn} \Leftrightarrow \Theta \text{ radians}$, so that $1 \text{ radian} \Leftrightarrow 1 / \Theta \text{ Turn} \approx 16\% \text{ Turn}$. Turns are measured on the Angular Circle and radians on the Unit Circle. See the earlier weblog entry. ⁸⁶ I suppose that mathematicians enjoy taking the ratio arc / radius, and then create a bit of mystery, while engineers directly use the Unit Circle, with $r = 1$ in the standard unit of measurement (meter, foot), with the magic of being practical without the mystery.

Dimensional analysis generally concerns the units such as meters and seconds and dollars, e.g. see here ⁸⁷ or on wikipedia ⁸⁸ again. I have been using it to good effect since

⁸⁴ <http://en.wikipedia.org/wiki/Radian>

⁸⁵ http://en.wikipedia.org/wiki/Dimensionless_number

⁸⁶ <https://boycottholland.wordpress.com/2014/07/07/why-are-radians-not-more-natural-than-any-other-angle-unit-q/>

⁸⁷ <http://hyperphysics.phy-astr.gsu.edu/hbase/units.html>

⁸⁸ http://en.wikipedia.org/wiki/Dimensional_analysis

early university, in particular since F.J. de Jong ⁸⁹ at Rijksuniversiteit Groningen had increased awareness there. In this case we apply dimensional analysis to our 2D space. I met a mathematician who thought that I thought that dimensional analysis applied only to 1D, 2D, 3D, ... and who started lecturing me, and it continues to amaze me how easy it is that misunderstandings arise.

Another possible misunderstanding is this. If you take a circumference of an object in 2D, say an equilateral triangle with sides 1 meter giving a circumference of 3 meter, and divide that circumference by a side, then it is conventionally (3 meter) / (1 meter) = 3 dimensionless, but rather be aware of (3 meter around) / (1 meter straight) = 3 around / straight. Again it is 2D / 1D = 1D. Just like the circle, you can make a turn going around that triangle. As it stands, it is little use to make an issue of this for circumferences in general, and the conventional view has its advantages. But for the circle it is useful to bring it to the fore in the definition of angle and turn. Indeed, here we need it to get the polar co-ordinates {radius, angle}, ⁹⁰ which uses that Turn is a separate dimension indeed.

⁸⁹ <http://www.dwc.knaw.nl/biografie/pmknaw/?pagetype=authorDetail&ald=PE00001124>

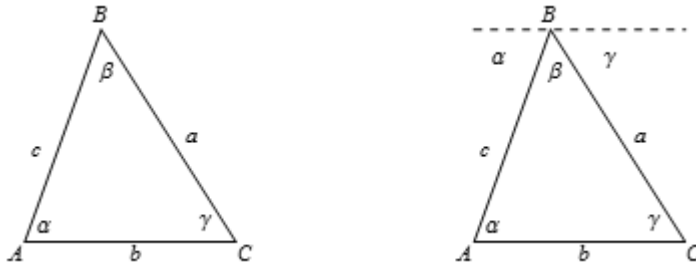
⁹⁰ http://en.wikipedia.org/wiki/Polar_coordinate_system

Euclid's fifth postulate

2014-06-29⁹¹

Judith Grabiner has a fine book *A historian looks back. The Calculus as Algebra and Selected Writings*, MAA 2010,⁹² in which she explains how Joseph-Louis Lagrange (actually Italian Giuseppe Lodovico Lagrangia, 1736-1813)⁹³ developed the derivative as algebra rather than with infinitesimals and limits. His method is more complex than my proposal in *Conquest of the Plane* (COTP) but his intuition is great.

Grabiner also explains how Lagrange wanted to get rid of Euclid's fifth postulate. This is equivalent to the property that a point can have only one line parallel to a given other line. The postulate is used to show that the sum of angles in a triangle add up to 180 degrees. With respect to the following diagram: take a line through B that is parallel to AC , and see how the angles α and γ are mirrored, so that $\alpha + \beta + \gamma = 180$ degrees.



Euclid: The sum of angles in a triangle is a half plane.

This latter proof is very elegant but also creates a perpetual wonder: Howcome do the angles in a triangle cover a half plane ? And why does this depend upon the fifth postulate that caused so much discussion ?

I always wondered whether it might be explained a bit clearer. Perhaps not as elegant but with faster acceptance and better retention. Let us try to see whether the fifth postulate can be replaced by another one with seemingly less dramatic portent.

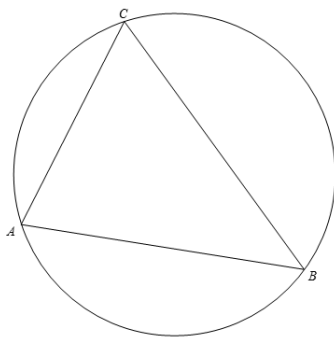
Deduction

Three points not on a line define a circle. Alternatively any triangle can be enclosed by a circle.

⁹¹ <https://boycottholland.wordpress.com/2014/06/29/euclids-fifth-postulate/>

⁹² <http://www.maa.org/publications/books/a-historian-looks-back>

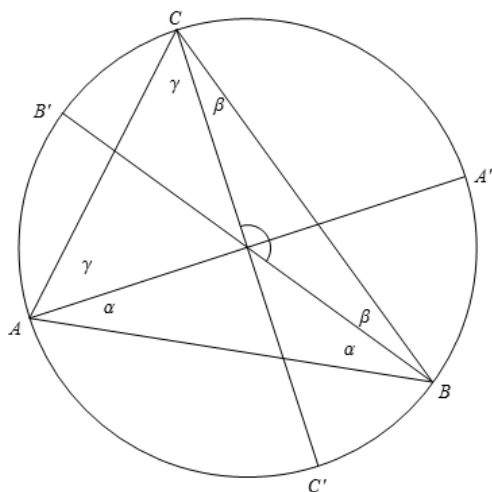
⁹³ http://en.wikipedia.org/wiki/Joseph-Louis_Lagrange



A circle can be defined by three arbitrary points not on one line. Any triangle can be enclosed by a circle.

We can scale triangle and circle up or down to the angular circle with its center at the origin $O = \{0, 0\}$ and circumference 1.⁹⁴ The unit of measurement of angles now is the plane itself. For example a value of a $1/2$ means a half plane or a half turn. The angular circle thus has radius $r = 1 / \Theta$, where $\Theta = 2 \pi$ and is pronounced 'Archi' from Archimedes. (Check that its circumference $\Theta r = 1$.)

The angles of a triangle seem to completely exhaust the angular circle. However, angles are measured from the center of the angular circle. Let us draw the diameters from the corners through the center, which gives AA' , BB' and CC' . We use the letters α , β and γ now for different angles.



Scaled to the angular circle. Diameters drawn through the corners.

There arise three inner isosceles triangles that use the same radius. The corner at A has angle $\angle BAC = \alpha + \gamma$. This angle on the circumference associates with $\angle BOC$ at the center (indicated by a tiny arc-sign) with the proper value $\angle BOC = \text{Arc}[B, C]$. Similarly for the other corners B and C .

⁹⁴ <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

For the angles at the center we find $\angle AOB + \angle BOC + \angle AOC = \text{Arc}[A, B] + \text{Arc}[B, C] + \text{Arc}[C, A] = 1$.

Since this can be done for any triangle, we arrive at the following postulate:

(*) For arbitrary corner A there is a proportion f so that $\angle BAC = \alpha + \gamma = f * \angle BOC = f * \text{Arc}[B, C]$

(1) It follows that the sum of the angles in a triangle equals that proportionality factor f too.

$$\angle BAC + \angle ABC + \angle ACB = f(\angle BOC + \angle AOC + \angle AOB) = f.$$

This also gives $\angle BAC + \angle ABC + \angle ACB = 2(\alpha + \beta + \gamma) = f$.

(2) Secondly, we can apply the newly found sum rule to the inner triangles too.

For $\triangle AOB$ we find $\angle AOB + 2\alpha = f$

For $\triangle BOC$ we find $\angle BOC + 2\beta = f$

For $\triangle AOC$ we find $\angle AOC + 2\gamma = f$

Adding these we find $1 + 2(\alpha + \beta + \gamma) = 3f$.

With the above: $1 + f = 3f$, or $f = 1/2$.

Combining (1) and (2):

(3) The sum of angles in a triangle is $1/2$.

(4) An angle that lies on the circumference of a circle is $1/2$ of the associated angle at the center of the circle.

For example: You may check for $\triangle AOB$ that we find

$$\angle AOB + 2\alpha = \text{Arc}[A, B] + f(\text{Arc}[B, A'] + \text{Arc}[A, B']).$$

Using that $\text{Arc}[B, A'] = \text{Arc}[A, B']$ we find that

$$\angle AOB + 2\alpha = \text{Arc}[A, A'] = \text{Arc}[B, B'] = 1/2 \text{ (a half plane indeed).}$$

Discussion

This proof strategy has these advantages:

(i) It emphasizes the measurement of angles, originally by plane sections but replaced by equivalent arcs. It shows that the angular circle is a natural way to measure angles. The 360 degrees came about historically because of the 365 days in the year but the plane itself makes more sense as a unit. (While 360 allows easy calculation: now use percentages: $1/2 = 50\%$.)

(ii) It shows clearly where the factor $f = 1/2$ comes from. There is a neat distinction between angles on the circumference and the actual measurement at the center.

(iii) Reversing the equivalences, we now have an elegant proof that a point has only one line parallel to another given line. Euclid's fifth postulate has become dependent upon (*).

(iv) Non-Euclidean geometry arises from adapting (*). When the axioms are applied to a sphere then a constant $f = 1/2$ doesn't make sense. It depends upon the kind of non-Euclidean geometry what the replacement postulate would be.

(v) Lagrange attached value to this discussion because scientists up to Einstein took Euclidean space also as a model for space itself. Grabiner op. cit. p261 suggests that ancient geometry was "the study of geometric figures: triangles, circles, parallelograms, and the like, but by the eighteenth century it had become the study of space (...ref...)."

Her reference is to Rosenfeld 1988 ch 5.⁹⁵ The implication would be that *The Elements* wouldn't be a study of space as well? I find this hard to believe – though I didn't read that reference. It would seem to me that Euclid already thought that he axiomised a theory of space. A theory of geometric figures and their properties would not make sense if they were not conceptualised as being in space. That the theory implied ideas about space (like: a finite line might have any length) would be so obvious that it wouldn't need mention. A fish in water would not speak about it. *The Elements* clearly isn't a kosmology like the *Timaeus*.⁹⁶ Given the importance of Plato, Euclid et al. likely regarded their findings limited to known space below the higher spheres, and they didn't need to speculate like Plato on what lay beyond. This mental set-up still implies a theory of (known) space. The shift in the 18th century likely would be that Plato's speculative kosmology fell away, so that Euclid's known space started to apply to the whole universe. Be that as it may be, nevertheless, let me refer to COTP p195-197 for a discussion that Einstein might have been a bit 'off' on the issue of measurement error. It may well be that Euclid's axioms actually *define* our very notion of space. At least, I find it impossible to think in terms of a 'curved space' – i.e. I can imagine a sphere only as an object within Euclidean space.

(vi) Let us return to the selection of postulates. Euclid's approach might well be better than the alternative given here. The postulate of a single parallel line feels rather natural. The proof on the triangle is so elegant that it may well have highest impact. However, Euclid's set-up is that the master selects the postulates and that we pupils follow his results. Nowadays we might adopt more daring didactics. (a) Indeed, start with the fifth postulate and use the elegant proof that the angles of a triangle add up to a half plane. Allow for the sense of wonder. (b) Discuss the alternative approach of assuming a constant proportional factor f , as above. (c) Discuss advantages and disadvantages. Then allow pupils their own choice. Indeed, this didactic structure has been used in this weblog entry. (d) Finally, clean up the mess. (1) In the first triangle, the corners were labelled clockwise to get the Greek letters in alphabetical order. The subsequent triangles have been properly labelled counterclockwise. (2) It would have been stricter if the isosceles triangles used indexed labels α_1 and α_2 but I opted for legibility. (3) Discuss the actual fifth postulate that Euclid used. Perhaps the original discussion about it was caused merely because of its needless complex format.

⁹⁵ <http://link.springer.com/book/10.1007%2F978-1-4419-8680-1>

⁹⁶ http://en.wikipedia.org/wiki/Timaeus_%28dialogue%29

The danger of complex number i

2014-10-15 ⁹⁷

The complex number $i = \sqrt{-1}$ has a danger that some people may not be aware of. We use $H = -1$, see here. ⁹⁸

For, consider:

$$-1 = i^2 = (\sqrt{H}) (\sqrt{H}) = \sqrt{(H H)} = \sqrt{1} = 1$$

Professor of mathematics Edward Frenkel ⁹⁹ states in his book, intended for the general audience, and thus giving false information to that general audience:

"Note that it is customary to denote $\sqrt{-1}$ by i (for "imaginary"), but I chose not to do this to emphasize the algebraic meaning of this number: it really is just a square root of -1 , nothing more and nothing less. It is just as concrete as the square root of 2 . There is nothing mysterious about it." (E. Frenkel, "Love & Math", p101-102)

Observe the factual error and the error in didactics:

1. The factual error is to say that the symbol $\sqrt{}$ has the same meaning in $\sqrt{-1}$ as in $\sqrt{2}$.
2. Didactically, it is writing i that conveys the algebraic meaning better, not $\sqrt{-1}$.

It took William Rowan Hamilton (1805-1865), ¹⁰⁰ the hero of Irish mathematics, a major part of his time to discover that $i = \{0, 1\}$, i.e. the point in the two-dimensional plane where $x = 0$ and $y = 1$. Stepping into another dimension is not the same as staying in the same dimension. If you treat those as the same then you get above deduction that $-1 = 1$. The conclusion is that i is an operator and not a common number. The step $(\sqrt{H}) (\sqrt{H}) = \sqrt{(H H)}$ is forbidden since it concerns an operator, with a different rule for $\sqrt{}$. We can only call i a '(complex) number' if we adapt the notion of 'number' to include it.

Let us look a bit more at the reason why i was mysterious and imaginary. Consider the quadratic equation, and let us 'complete the square' on the left hand side

$$a x^2 + b x + c = 0 \quad \text{(formula for a vertical parabola)}$$

$$x^2 + b a^H x = -c a^H \quad \text{(bring } c \text{ to the right and multiply by } a^H = 1/a)$$

$$(x + b a^H 2^H)^2 = (b a^H 2^H)^2 - c a^H \quad \text{(using } 2^H + 2^H = 1)$$

$$x + b a^H 2^H = \pm \sqrt{(b a^H 2^H)^2 - c a^H} \quad \text{(discriminant)}$$

$$x = a^H 2^H (-b \pm \sqrt{(b^2 - 4 a c)}) \quad \text{(the quadratic formula)}^{102}$$

From wikipedia: this formula covering all cases was found by Simon Stevin in 1594, who also gave us the decimal dot. The present form was given by Descartes in 1637. In the past people were calculating every step. Having the final formula allows you to reduce the actual number of calculations you have to do.

⁹⁷ <https://boycottholland.wordpress.com/2014/10/15/the-danger-of-complex-number-i/>

⁹⁸ <https://boycottholland.wordpress.com/2014/08/30/taking-a-loss/>

⁹⁹ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

¹⁰⁰ http://en.wikipedia.org/wiki/William_Rowan_Hamilton

¹⁰¹ <http://en.wikipedia.org/wiki/Parabola>

¹⁰² http://en.wikipedia.org/wiki/Quadratic_equation

There will be an intersection with the horizontal axis (above equation has a root) only if $D \geq 0$. Otherwise there is no intersection.

It is an option to interpret $i = \sqrt{H}$ as a number too. In that case the problem is redefined to have existed in the complex plane all along, and then there is always a solution. This explains where the mystery comes from: you have to grow aware that your original problem was not one-dimensional but two-dimensional.

Frenkel's approach *'there is nothing mysterious about it'* kills this last insight. He claims to draw you to the beauty of mathematics, comparable to masterpieces of art, but at the same time he says that you should not be worried since it is as common as bread and butter. There is a difference between admiring a masterpiece and making one yourself. The professor is seriously confused. It is better that students understand the quadratic equation and the complex plane, and then admire their own understanding too.

(Graphics: Parabolic jump (Source: Jarek Tuszynski, wikimedia commons))

Calculus and dynamic division

Calculus

2013-11-13 ¹⁰³

The Economist November 9 2013, p45: ¹⁰⁴

"Bill Gates once said that if every child had mathematics teachers as good as those in the top quartile, the achievement gap between America and Asia would vanish in two years. (His lecture has been watched 1.5 m times online.)"

In fact, listen to "*How do you make a teacher great ?*" TED 2009. ¹⁰⁵ It is quite a bold claim but there is some element of truth in it, while it helps to be so ambitious. While you are at it, perhaps also consider the Wall Street Journal interview ¹⁰⁶ and the TED 2013 lecture ¹⁰⁷ that teachers need coaches and feedback (other than from students).

The same November 9 2013, I had a presentation in workshop D5 of the annual *Dutch Teachers of Mathematics Day*. ¹⁰⁸ My presentation was on the algebraic approach to the derivative, see the slides, ¹⁰⁹ the YouTube lecture ¹¹⁰ and the book *Conquest of the Plane*.

Some mathematicians are aware of an algebraic content in the derivative, see for example here, ¹¹¹ but they still stick to the use of limits. The new notion of 'dynamic division' removes the need for limits and greatly simplifies and clarifies calculus.

The discussion on the derivative links up with my earlier blog ¹¹² on the mathematical constant Archi = $\Theta = 2 \pi$. The derivative generally gives the slope of a function, but the slope is also the tangent given by the sine / cosine ratio that depends upon the angle.

Mathematical teaching requires re-engineering. Much in the highschool programme can be made more accessible to students. Society can advance when people have a better understanding of logic, finance, voting theory, and the distinction between model and reality.

My original presentation was in Dutch and was split up in two sections of 15 minutes, with 5 minutes inbetween for questions and 20 minutes afterwards for discussion. The reception was OK, with serious questions and discussion, and modest applause at closure. Don't expect teachers of mathematics to jump and dance on tables. It will take a lot of time before the new algebraic approach will be used in the schools and make math clearer for students. A comment by one of the teachers: Holland cannot change by itself, we have to stick to the world standard. My suggestion is that when math teaching is improved, so that students better understand math, then it should not be too difficult to explain what the world standard is and why it is so crummy.

¹⁰³ <https://boycottholland.wordpress.com/2013/11/13/calculus/>

¹⁰⁴ <http://www.economist.com/news/united-states/21589427-states-are-starting-test-teachers-your-marks>

¹⁰⁵ http://www.ted.com/speakers/bill_gates.html

¹⁰⁶ <http://online.wsj.com/news/articles/SB10001424053111903554904576461571362279948>

¹⁰⁷ http://www.ted.com/talks/bill_gates_teachers_need_real_feedback.html

¹⁰⁸ <http://www.nvww.nl/page.php?id=9496&rid=597>

¹⁰⁹ <http://thomascool.eu/Papers/COTP/2013-11-11-ColignatusStudiedagNVWW-English.pdf>

¹¹⁰ http://youtu.be/gn_BKZaDa-o

¹¹¹ http://people.hofstra.edu/stefan_waner/realworld/tutorials/frames2_4.html

¹¹² <https://boycottholland.wordpress.com/2012/02/18/mathematical-constant-archimedes/>

The video that I put on YouTube uses English and lasts 47 minutes. Thus it is not in the TED 20 minutes mold. The discussion in Dutch wasn't recorded so I included an additional quarter of an hour to discuss some points from that discussion. The sound recording is bad, I should get a better microphone. I practiced in Dutch and not in English, so the presentation is not as fluent as it could be. I am already aware of this, so there is no need for *feedback* on these aspects ...

Slander squared

2014-06-12 ¹¹³

When you are mugged and call a policeman, and when that policeman protects the mugger, then you feel abused in squared fashion. I greatly enjoy the Jack Vance SF novels that project 19th century scoundrel stories into the far future, but it is quite a difference to be abused yourself here and now.

The book *Conquest of the Plane* (COTP, 2011) re-engineers mathematics education. It uses the critique on traditional mathematics education given in *Elegance with Substance* (EWS, 2009). The PDFs are on their websites.

Two book reviewers of EWS and/or COTP advised to read COTP with an open mind. A “review” by Jeroen Spandaw (TU Delft) however misrepresents the analysis and is slanderous. Spandaw rejected my suggestion to talk about it.

A subsequent appeal to the TU Delft Commission on Scientific Integrity resulted in their decision in 2012 that book reviews are not at issue in the rules on scientific integrity, or alternatively that the supposed breach was so minor that it didn’t justify an effort to look deeper into the matter (with an actual investigation).

An appeal to the national supervising integrity body LOWI of the Royal Dutch Academy of Sciences KNAW confirmed this Delft verdict in 2014.

A report by Christiaan Boudri (Arnhem-Nijmegen) in 2013 was published after my appeal at TU Delft and before my appeal to LOWI. It reacted to Spandaw with the repeat advice to read with an open mind. The two integrity committees regarded this as too late for the original appeal at TU Delft, whence it had no impact, even though it was in time for the LOWI appeal.

It is amazing that the committees on integrity of science think that reviews of scientific books are not part and parcel of science. When scientists discuss the works of other scientists (listed in their references) then there are standards of fair representation and common decency. Why would those standards not apply to book reviews as well ?

Spandaw’s “review” is in Dutch. My discussion and protest of June 11 2014 is in English:

Reviewing a scientific book isn’t science ?

“Conquest of the Plane” and scientific integrity versus misrepresentation and slander ¹¹⁴

At no time the freedom of expression of a scientist is at issue here. If Spandaw is not convinced by COTP he is free to say so. At issue is only that you don’t misrepresent and slander and you don’t accept it from others.

The best approach remains that others read EWS and COTP. It is unfortunate that there now is the added burden to have to think about whether you support the misrepresentation and slander or not. All this might perhaps be beneficial for the discussion on mathematics education. Mathematics education might be better than in the year 0 but is rather dismal compared to what is possible. More discussion of EWS and COTP will help to get an improvement.

Traditional mathematics educators like Spandaw think that they defend quality but they close their minds to the wonderful results that are possible when we re-engineer the traditional lores that we teach our students.

¹¹³ <https://boycottholland.wordpress.com/2014/06/12/slander-squared/>

¹¹⁴ <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

Part 2.

Breach of integrity in Holland

Integrity of science

For IMU / ICMI: Integrity of science in Dutch research in didactics of mathematics

2014-09-02 ¹¹⁵

To: Professor I. Daubechies ¹¹⁶
From: Thomas Cool / Thomas Colignatus
Subject: For IMU / ICMI: Integrity of science in Dutch research in didactics of mathematics
Cc: secretary of the IMU, president of KWG, professor Andre Ran

To the president of the International Mathematical Union (IMU), ¹¹⁷ that has the International Commission on Mathematical Instruction (ICMI) ¹¹⁸

Dear professor Daubechies,

My email of July 16 can be updated integrally as follows, and I will put this present email on my weblog.

Let me invite you to read these two weblinks:

<https://boycottholland.wordpress.com/2014/07/16/integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

<https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

Let me invite you to also read this paper: "Pierre van Hiele and David Tall: Getting the facts right" (version 2, 2014-08-30) at <http://arxiv.org/abs/1408.1930>.

Let me invite you to keep the matter on your desk with priority, involve others in IMU / ICMI who could advise you on this, and aim at board decisions that result into proper resolution.

Since this email concerns research in didactics, your tendency would be to forward it to ICMI. My suggestion is not to give them total freedom but set up an overall IMU committee to monitor the process within ICMI on this. In itself it might be proper to hand the issue to ICMI, since when they succeed in resolving the issue, then it would meet with greater acceptance in their own circles. On the other hand, there will be a tendency to reject criticism. Hence my suggestion to keep the issue on your desk as well.

One of the problems is that ICMI has a "Hans Freudenthal Award / Medal", ¹¹⁹ which indicates that ICMI has not been able to detect the fraudulent nature of Freudenthal's "research" and appropriation of ideas of Pierre and Dieke van Hiele. A related problem is that the Dutch representative to ICMI might not have transferred my earlier message on didactics in general.

¹¹⁵ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics>

¹¹⁶ http://en.wikipedia.org/wiki/Ingrid_Daubechies

¹¹⁷ <http://www.mathunion.org/>

¹¹⁸ <http://www.mathunion.org/icmi>

¹¹⁹ <http://www.mathunion.org/icmi/activities/awards/introduction/>

Since your background is Belgian, I presume some knowledge of Dutch, and then let me also directly include the link to my letter to KNAW-LOWI, which is the Integrity of Research department of the Dutch Royal Academy of Sciences:
<http://thomascool.eu/Papers/Math/2014-07-08-Colignatus-aan-KNAW-LOWI.html>

I imagine that IMU might not have the resources available at KNAW-LOWI. My suggestion is that IMU supports my suggestion to KNAW-LOWI to look into this, even though they have already declined my original suggestion. In that case I would hope that there is international monitoring of the investigation at LOWI too, since they might be less critical on what went wrong in Holland. There are some issues here, some of which seem quite local but that still would greatly benefit from international monitoring: (1) the habit of abstract thinking mathematicians and such teachers to forget about the real world and empirical methods, (2) Hans Freudenthal and his “work” (much in Dutch), (3) the renaming of the ICMI Award, say to a “Piaget & Van Hiele Award / Medal”, (4) the abolition of the Dutch “Freudenthal Head in the Clouds Realistic Mathematics Institute” (FI = FHCRMI) here in Holland. It would seem that the last would not be in the ballpark of IMU but it is important to be aware that the institutional drive of that institute is to defend Freudenthal’s “legacy”, and thus to oppose criticism on the other points too, at the detriment of IMU / ICMI. It is better to be straightforward on the logic from the outset, and have international monitoring.

I alerted the Presidents of MAA (Bob Devaney) and AMS (David Vogan) and the director of the US Institute of Education Sciences (IES, John Easton)¹²⁰ on the two weblog links, but not on my recent paper on Van Hiele and Tall (yet). After putting this letter on my weblog, I will alert the board of NCTM (Diane Briars) to this.¹²¹ I now copy to the IMU secretary and the chair of the Dutch KWG, now professor Geurt Jongbloed but formerly Andre Ran. In my perception KWG has been seriously failing on this issue but if IMU would indicate that there is an issue indeed then they might perhaps be willing to help out, with some international monitoring.

My position in all of this is quite limited, and mainly described in my books “Elegance with Substance” (2009) and “Conquest of the Plane” (2011) and the Dutch “Een kind wil aardige en geen gemene getallen” (2012), see my website, where the PDFs of the first two can be found. I do not claim particular expertise on Freudenthal’s “work” but what I have read didn’t appear so practical, except for what he took from Pierre van Hiele. I am amazed both by RME’s adoption for education and the lack of interest to repeal it now that everyone can see that it doesn’t really give results, except for the part taken from Van Hiele. My main point is no 1 above: the habit of abstract thinking mathematicians and such teachers to forget about the real world and empirical methods. My suggestion is that we need “engineers in education” rather than such mathematicians, and that education requires the “medical school” model in which education and its research are attuned, see this other link:

<https://boycottholland.wordpress.com/2014/07/18/the-medical-school-as-a-model-for-education/>

Sincerely yours,

Thomas Cool / Thomas Colignatus Econometrician (Groningen 1982) and teacher of mathematics (Leiden 2008) Scheveningen, Holland
<http://thomascool.eu/>

PM. Readers of this email may also be interested in: Elizabeth Green: *Why Do Americans Stink at Math?*, NY Times July 23 2014¹²²

¹²⁰ <http://ies.ed.gov/aboutus/>

¹²¹ <http://www.nctm.org/about/default.aspx>

¹²² http://www.nytimes.com/2014/07/27/magazine/why-do-americans-stink-at-math.html?_r=1

Freudenthal's "realistic mathematics education" appears to be a fraud

2014-07-06 ¹²³

Sitting on the bench in the park where he died, I hope that Hans Freudenthal whispered: "Forgive me, Pierre, for what I have done to you."

Here we follow the Amir Alexander method of first selecting the storyline and then fill in the data. ¹²⁴ In this story Hans Freudenthal (1905-1990) ¹²⁵ is the crook and Pierre van Hiele (1909-2010) ¹²⁶ is the hero. It might be that Freudenthal is no real crook ¹²⁷ but never waste a good story. However, Pierre van Hiele remains the hero for a fact. The storyline is given by this table.

<i>Useful for mathematicians, likely higher up in the autistic spectrum</i>	<i>Useful for nobody</i>	<i>Useful for education of scientists and the general public</i>
Euclid 300 BC ¹²⁸		
New Math, ¹²⁹ after Sputnik 1957 ¹³⁰		1957 theses Pierre van Hiele under Freudenthal and Dian Geldof under Langeveld (Utrecht archive, ¹³¹ math genealogy ¹³²)
Hans Freudenthal's earlier work on mathematics (topology, assistant to L.E.J. Brouwer)	Hans Freudenthal on education after 1957. His "realistic mathematics education" that isn't realistic	

The education in mathematics had been dominated by Euclid's *Elements*. Admittedly, Newton and Leibniz added some aspects on the derivative, but that is small beer in the shadow of the great Greek. In 1957 Russia launched its Sputnik and America woke up to the reality that their system of education didn't produce enough rocket scientists. Teachers of mathematics rushed in to assist with the New Math curriculum. However, in 1973 Morris Kline wrote *Why Johnny Can't Add: the Failure of the New Math*. The New Math was much too abstract and actually quite silly. Hans Freudenthal rushed in with his "realistic mathematics education". His "realism" looked like the proper answer to the earlier abstractions, and his "guided re-invention" sounded like that every child would reinvent mathematics if guided properly by its math teachers. Nowadays, the *International*

¹²³ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

¹²⁴ <https://boycottholland.wordpress.com/2014/06/14/amir-alexander-and-history-as-storytelling/>

¹²⁵ http://en.wikipedia.org/wiki/Hans_Freudenthal and <http://www-history.mcs.st-and.ac.uk/Biographies/Freudenthal.html>

¹²⁶ http://en.wikipedia.org/wiki/Van_Hiele_model

¹²⁷ This statement was written in early July 2014. Later there is <http://arxiv.org/abs/1408.1930>

¹²⁸ <http://en.wikipedia.org/wiki/Euclid>

¹²⁹ http://en.wikipedia.org/wiki/New_Math

¹³⁰ http://en.wikipedia.org/wiki/Sputnik_1

¹³¹ <http://dap.library.uu.nl/frames.html?zoeken>

¹³² <http://genealogy.math.ndsu.nodak.edu/id.php?id=102372>

Commission on Mathematics Instruction (ICMI) ¹³³ has the Freudenthal Medal, ¹³⁴ as if this should be something to be proud of, and not a disgrace to mankind.

The point namely is that Freudenthal was an abstract thinking mathematician too. His “realism” is an abstract kind of “reality”. His invention of this “realism” hadn’t had guidance by real teachers with practical experience. When Freudenthal spoke about statistics he meant mathematical statistics, and he didn’t like it anyway. In his own teaching, he tended to bully his students and when they skipped his course he seemed to regard it as an admission of stupidity – see this article ¹³⁵ in the Dutch journal of mathematics NAW. David Tall has the story that when he hadn’t met Freudenthal yet but criticised him then he became as nice as can be, which flip-flop behaviour is rather telling, while that story doesn’t tell whether Freudenthal actually corrected his mistakes.

David Tall (2006, top p2): ¹³⁶ “When Richard [Skemp] was asked to review Freudenthal’s book *Mathematics as an Educational Task* [1973], having already bought his own copy and not wanting another, he passed the invitation to me. To review a work of the great Freudenthal was a huge task for a young mathematics lecturer and I sought advice from a senior colleague, James Eels, who knew him well. He confirmed that I should say exactly what I felt and, emboldened by his advice, I wrote a welcoming but critical essay. I received a post-card from Freudenthal after the review [1975] ¹³⁷ appeared: “thank you for the review which I enjoyed, especially the critical parts.”” (Quote added July 11 2014) (July 8 2014: See the newly included *Appendix* below with an email by David Tall who thinks that he corrects me but he doesn’t.)

Apart from his lunch, Freudenthal had two contact points with reality. He did some work on the history of mathematics, and it is up to the historians of mathematics to check whether it is realistic, rather than to assume that it can be safely referred to. The real historian is B.L. van der Waerden (1903-1996), ¹³⁸ see this fine interview at AMS. ¹³⁹ Also, Freudenthal supervised the thesis by Pierre van Hiele on the education of mathematics, and read Dina Geldof too. In his memoirs Freudenthal tells that when he found his mathematical abilities waning, he had the choice to continue with history or education of mathematics. Perhaps it was good for the history of mathematics that he chose education.

In itself it is remarkable that he didn’t resign from the chair of professor of mathematics and switched to the education in mathematics but could proceed as professor. What should have been proper too is that, if he really wanted better math education, then he should have helped Pierre van Hiele to be come professor in the education of mathematics, so that Van Hiele with his empirical ways could teach new students.

In the theses, Van Hiele & Geldof had identified levels of understanding of mathematics. The lowest level is in the realm of the senses. Seeing, tasting, touching, weighing, pushing and so on. In need of a theory of his own, Freudenthal referred to this as “realistic mathematics education”.

Freudenthal did refer to Van Hiele on occasion. Once he had his own publications, it was easier to refer to those, so that Van Hiele passed into oblivion. At conferences people were surprised that he still was alive. Van Hiele remained a highschool teacher for the rest of his life but used his experience to write about the education in mathematics. In sordid manner the University of Utrecht now has a “Freudenthal Institute” for the

¹³³ <http://www.mathunion.org/icmi/home>

¹³⁴ <http://www.mathunion.org/icmi/activities/awards/the-klein-and-freudenthal-medals/>

¹³⁵ <http://www.nieuwarchief.nl/serie5/pdf/naw5-2010-11-1-054.pdf>

¹³⁶ <http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2006b-prague-journey.pdf>

¹³⁷ <http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1977b-review-freudenthal.pdf>

¹³⁸ http://en.wikipedia.org/wiki/Bartel_Leendert_van_der_Waerden

¹³⁹ <http://www.ams.org/notices/199703/interview.pdf>

education in mathematics and science, and Van Hiele has a page in their wiki.¹⁴⁰ (NB: August 18: An error there says that Pierre wrote his thesis under Langeveld.) Here is an interview¹⁴¹ with Van Hiele in the Dutch journal of mathematics, that shows his disgust of Freudenthal with his partly stolen good and otherwise crooked ideas on education. Van Hiele remains civilized though, so typical of the good math teacher.

My book *Conquest of the Plane* is based upon Pierre van Hiele's full approach. My book also explains why the view by Freudenthal was wrong, and how Freudenthal mistook his own mathematical abstractions for realism.

Holland now has the problem that whoever in Holland proves that Freudenthal was a fraud, is slaughtered. The proof doesn't count. What counts is the indoctrination from Utrecht. What counts is the need to treat each other politely and not delve skeletons from the closet. What counts is the international great reputation of Freudenthal. What counts is that Dutch mathematicians are afraid of having to admit that they never really look at the education of mathematics but were bullied by Freudenthal.

On June 30, the Dutch Akademy of Sciences (KNAW) in Amsterdam had a conference on the education of arithmetic. Here is my report of the event¹⁴² (unfortunately in Dutch). The conference was an intellectual embarrassment and failure, even though it was led and fed by internationally reputed mathematician Jan Karel Lenstra.¹⁴³ The key problem is that Lenstra is another abstract minded mathematician who has no clue about the empirical nature of education. There are also issues of policy. There are 150,000 teachers at Dutch elementary schools that require re-education on teaching mathematics including arithmetic, but the state secretary chooses politically to leave them be, and let the problems be solved by the 4,000 teachers of mathematics in secondary education. Admittedly new teachers should be trained better during their education, but the annual batch of new teachers will have not much effect. Professor Lenstra supports the state secretary on this policy, instead of reporting accurately that this policy will wreak havoc.

It is an international problem. Hans Magnus Enzensberger¹⁴⁴ wrote the book *The Number Devil*¹⁴⁵ to show children how arithmetic can be great fun. It is telling that teachers at elementary school apparently kill the fun. The latter of course cannot be blamed all on Hans Freudenthal but you will be surprised how much.

(picture: *The Number Devil*) (source: <http://www.mobygames.com>, screenshots) (Guess what the lesson is about.)

I move:

- that ICMI rebaptises the "Freudenthal award and medal" into the "Jean Piaget and Pierre and Dieke van Hiele award and medal"
- that the *Freudenthal Institute* is abolished and that a new education research institute is founded that used empirical methods, say the Simon Stevin Institute
- that everyone calls the present *Freudenthal Institute* by its proper name the *Freudenthal Head in the Clouds Realistic Mathematics Institute* to properly describe that their "realism" has no base in empirics, while also to get rid of the psychological connotations that a person who has an institute named after him ought to have had some great results

¹⁴⁰ http://www.fisme.science.uu.nl/wiki/index.php/Pierre_van_Hiele

¹⁴¹ <http://www.nieuwarchief.nl/serie5/pdf/naw5-2005-06-3-247.pdf>

¹⁴² <http://thomascool.eu/Papers/Math/2014-07-02-KNAW-Rekenen-deugt-niet.html>

¹⁴³ http://en.wikipedia.org/wiki/Jan_Karel_Lenstra

¹⁴⁴ http://en.wikipedia.org/wiki/Hans_Magnus_Enzensberger

¹⁴⁵ http://en.wikipedia.org/wiki/The_Number_Devil

- that KNAW supports my proposal that Dutch parliament does an enquiry in the education of mathematics, to determine what went wrong and what funds need to be made available for improvement
- that the world boycotts Holland till the censorship of economic science by the Dutch government is resolved, see above *About* page.¹⁴⁶

PM 1. Let me add that mathematics is an essential part of your life. It may be that the old education has spoiled this for you, but you could understand that you should try that this doesn't happen to your children and grandchildren.

PM 2. Let me also add that Jeroen Dijsselbloem¹⁴⁷ made his name in Dutch politics when he chaired a parliamentary enquiry into the educational failure of the "studiehuus".¹⁴⁸ The conclusion of his committee was that policy makers should determine *what* is taught, and teachers should determine *how*. See however the proper problem: Policy makers have determined that pupils should have mathematics, but the educators provide something that they call "mathematics" that it is not. While arithmetic scores of Dutch pupils might remain acceptable, there is much to say about testing, and, schools might put more time into language and arithmetic and less into history and music. Thus Dijsselbloem did not really get to the core of the issues. As chair of the Eurozone he is now doing the same with the European economy.

Appendix included on July 8 2014: David Tall's statement from his own memory

Since I referred above to a text that I only remembered from David Tall, I alerted him to this weblog entry and invited him to correct me if needed. I received the email below and on his request gladly include it. I consider it very important that the witnesses of the history of "realistic mathematics education" are heard on this. Professor Tall complains that this weblog doesn't allow comments. It is for the simple reasons of both legibility and my limited time to monitor responses. If people want to respond then it better be by reasoned exposition on their own websites. Tall also states that I misrepresent his position. I do not. It appears that I remembered his text correctly. The Dutch journal of mathematics article by Nellie Verhoef and Ferdinand Verhulst¹⁴⁹ quote Kalmijn (p55) stating that Freudenthal quickly divided students in dumb or smart, and did this even with government ministers. This may be exaggerated but gives an indication. Strooker reports: Classes were too hard for many but "if you passed you became a good mathematician". That particular interviewed person does not reflect whether more students might have passed with a less intimidating approach. I find this telling for a theorist of education. This should not be read as a caricature. Of course there are also interviews with a positive load, but the point is the news of the intimidation in education. Apparently Freudenthal did not intimidate David Tall, and Tall's direct criticism provoked a kind attitude from Freudenthal. This is telling for what I call flip-flop behaviour of treating some as idiots but becoming kind when being treated with criticism yourself. It is useful to have that link to Tall's text and historians may check whether Freudenthal corrected his texts.

My point remains, which is that Tall can only report a positive view on Freudenthal since he escaped the intimidation. Tall still does not seem to be aware of how much of intimidation he escaped. Having success in his own work might make it more difficult to see the students who needed a different approach in education and research. I move that Nellie Verhoef has the article translated into English so that professor Tall and others can read the reports themselves. Of course I reject professor Tall's qualifications "vindictive" and "diatribe". His evidence confirms the story, and he may not have the other evidence to reject his rosy view.

¹⁴⁶ <https://boycottholland.wordpress.com/about/>

¹⁴⁷ <https://boycottholland.wordpress.com/2013/08/23/dijsselbloem-on-dutch-exports/>

¹⁴⁸ <http://nl.wikipedia.org/wiki/Commissie-Dijsselbloem>

¹⁴⁹ <http://www.nieuwarchief.nl/serie5/pdf/naw5-2010-11-1-054.pdf>

I agree and already decided myself that the Pierre and Rian van Hiele theory applies to all area's of mathematical development, or indeed any developed discipline. This seems like a matter of logic and definition when finding the right terms to describe what (mathematical) learning is. Professor Tall presents "Realistic Mathematics" as "part of the long-term evolution of theories". This is complex. With Van Hiele, there is no need for "Realistic Mathematics Education". The good elements derive from Van Hiele, and the name is taken from the basic Van Hiele level. Thus, take a horse, have it breed with a donkey, call the result "hooves", and then try to argue that the infertile "hooves" is part of evolution. Of course I ought to return to the issue when I have read professor Tall's book. (August 27: see here. ¹⁵⁰)

I suppose that professor Tall writes "Thomas C" because of my use of "Colignatus" in science, but this is not how I am normally called.

Tue, 08 Jul 2014

Dear Thomas C,

Thank you for alerting me to your publication on the web. It is in the form of a blog which does not seem to admit a response, allowing you to put forward a discourteous and personalised attack on Freudenthal, including a misrepresentation of something I said to you, without the right to reply.

Your summary of my comments about Freudenthal are quoted out of context to serve your own purpose. The facts are these: In the seventies, as a young mathematician turned math educator with few publications to my name, I wrote a strong critical review of Freudenthal's work which is still available as a download my website:

<http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot1977b-review-freudenthal.pdf>

Looking at this review nearly forty years on, I am surprised at my own driven directness which clearly appealed to Freudenthal, who sent me a postcard which simply said 'Thank you for your review which I enjoyed very much, especially the critical bits'. Subsequently he was critical of my work in a very supportive way which I greatly respected. His driven personality was responsible for the introduction of mathematics education to the International Congress of Mathematicians and subsequently to the formation of ICME and then PME. He played a central role in the development of mathematics education as a discipline.

In my book on *How Humans Learn to Think Mathematically*, which you say you have yet to read, you will see Freudenthal mentioned only in a footnote and Realistic Mathematics considered in the final chapter as part of the long-term evolution of theories. My own perspective builds on the underlying commonalities of many theoretical frameworks from diverse disciplines. You may be surprised to know that the work of your heroes, the van Hieles, in a simpler but broader format, applies not only to geometry but to all areas of mathematical development that even Pierre van Hiele himself did not realise, and denied in print.

Theories evolve, and in that evolution, Freudenthal played a central role. It does you no credit to attack him in a personalised vindictive manner. Your diatribe does more damage to yourself and your cause than it does to Freudenthal.

If you have a sense of fairness, I suggest that you place this e-mail as a response on your blog page.

¹⁵⁰ <http://arxiv.org/abs/1408.1930>

I have sent a copy of this e-mail to Nellie Verhoef who, as you know, is closely acquainted with both Freudenthal and van Hiele, and is herself referred to indirectly in your blog.

David Tall

Addendum to this Appendix, July 15 2014: On Pierre van Hiele & Richard Skemp

In above email, professor Tall states: *"You may be surprised to know that the work of your heroes, the van Hieles, in a simpler but broader format, applies not only to geometry but to all areas of mathematical development that even Pierre van Hiele himself did not realise, and denied in print."*

Please do not mistake critical admiration for hero worship. But I found this surprising indeed, and as proof, Tall sent me a copy of Van Hiele's chapter in the book that he edited, the *Tribute to Richard Skemp* (2002).¹⁵¹ (Amazon¹⁵² and I have difficulty locating its publisher, but a review is here.¹⁵³)

However, the proof that Tall suggests isn't there. In that chapter, Van Hiele warns: p28:

"The problems in algebra that cause instrumental thinking have nothing to do with level elevation since the Van Hiele levels do not apply to that part of algebra. People applied terms such as 'abstraction' and 'reflection' to the stages leading from one level to the next. This resulted in a confusion of tongues: we were talking about completely different things."

Thus "*part of algebra*" should not be mistaken for *all* algebra.

p39: "The transition from arithmetic to algebra can not be considered the transition to a new level. Letters can be used to indicate variables, but with variables children are acquainted already. Letters can be used to indicate an unknown quantity, but this too is not new."

p43: "The examples Skemp mentions in his article about I2, R2 and L2 do not have any relations with a level transition. They are part of algebra in which topic, as I have emphasised before, normally level transitions do not occur." Again "*part of algebra*" should not be read as "algebra" as a whole.

In his final conclusion, p46: "In most disciplines there are different levels of thinking: the visual level, the descriptive level and the theoretical level." Indeed, Van Hiele gives such various examples in my copy of "Begip en Inzicht" (1973) which is more extended in English in "Structure and Insight" (1986).

Thus, Van Hiele was aware of the portent of his theory, contrary to what David Tall suggests. So much more of a pity that Freudenthal sabotaged and appropriated it. I will return to the issue when I have read professor Tall's book. If you can't wait, start reading *Conquest of the Plane*, section 15.2, p201-206.

Update July 28 2014: There is now my PDF article *Pierre van Hiele and David Tall: getting the facts right*.¹⁵⁴

¹⁵¹ <http://www.grahamtall.co.uk/skemp/tribute.html>

¹⁵² <http://www.amazon.co.uk/Intelligence-Learning-Understanding-Mathematics-Tribute/dp/1876682329>

¹⁵³ http://www.merga.net.au/documents/MERJ_16_1_Gough.pdf

¹⁵⁴ <http://thomascool.eu/Papers/Math/2014-07-27-VanHieleTallGettingTheFactsRight.pdf> or <http://arxiv.org/abs/1408.1930>

Integrity of science in Dutch research in didactics of mathematics

2014-07-16 ¹⁵⁵

I am sorry to report that Holland also fails on the integrity of science in the research on the didactics & education of mathematics. This is my letter ¹⁵⁶ (in Dutch) to the Scientific Integrity body LOWI of the Dutch Royal Academy of Sciences KNAW. ¹⁵⁷

Earlier, in my book *Elegance with Substance* (2009), I made the empirical observation that mathematicians are trained for abstraction while education is an empirical issue. The training of mathematicians to become teachers of mathematics apparently can often not undo what has been trained for before. This basically means that many have lost the ability to *observe*. Math teachers tend to solve their cognitive dissonance by adhering to “mathematical tradition” that however is not very didactic, and that in fact collects the didactic debris of past centuries.

A key example here in Holland is the difference between Hans Freudenthal ¹⁵⁸ as the abstract topologist and Pierre van Hiele ¹⁵⁹ who as a mathematician and actual teacher however kept his ability to observe. We need only look at the debris in math textbooks to observe that the majority of math teachers aren’t like Van Hiele. See *Elegance with Substance* if you cannot identify the debris yourself.

An international example on statistics is the difference between Fisher and Gosset on “statistical significance”. ¹⁶⁰ Mathematicians tend to consider mathematical statistics only, and are little aware of empirical significance. Math educators who nowadays use statistics might fall victim to ‘garbage in, garbage out’ but nevertheless be praised as ‘empirical’.

Now in 2014 that empirical observation comes with a sting. When abstract thinking mathematicians make statements about the empirical reality of didactics & education, they actually make statements out of their province, about something they haven’t studied: *which is a breach of research integrity*. This especially holds when they have been warned for this, say by my 2009 book (listed in the AMS Book List, Notices Vol 58, No 11, p1474), ¹⁶¹ or perhaps even directly by me.

In Holland there now is the case of internationally known Jan Karel Lenstra, ¹⁶² who did work in operations research, (linear) programming, scheduling and the traveling sales person, who was selected in 2009-2010 by theoretical physicist and KNAW President Robbert Dijkgraaf ¹⁶³ to chair a KNAW “Committee on Mathematics in Primary Education”. The Dutch complaint is that children don’t learn arithmetic so well anymore, e.g. aren’t trained on long division as a sure method. It often happens that a committee is chaired by a person who doesn’t know much about the subject beforehand, but then that person tends to be aware of this, and is willing to learn. In the case of Lenstra, he apparently thought that he knew enough about “Mathematics in Primary Education” so that he also understood the didactics & education itself.

Hans Freudenthal had a huge impact on arithmetic in Dutch primary education. Lenstra observes about the Freudenthal madness:

¹⁵⁵ <https://boycottholland.wordpress.com/2014/07/16/integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

¹⁵⁶ <http://thomascool.eu/Papers/Math/2014-07-08-Colignatus-aan-KNAW-LOWI.html>

¹⁵⁷ https://www.knaw.nl/en?set_language=en

¹⁵⁸ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

¹⁵⁹ http://en.wikipedia.org/wiki/Van_Hiele_model

¹⁶⁰ http://www.deirdremccloskey.com/articles/stats/preface_ziliak.php

¹⁶¹ <http://www.ams.org/notices/201110/rxx111001471p.pdf>

¹⁶² <http://www.cwi.nl/people/275>

¹⁶³ <https://boycottholland.wordpress.com/2012/03/08/robbert-dijkgraaf-as-darth-vader/>

“The core is that we must get more evidence-based research [education ?]. The ‘realistic arithmetic’ [RME] has been adopted without the empirical evidence to make it obvious. And also the PABO [training of elementary school teachers] has been constituted on the base of beliefs instead of scientific research.” (my translation, comments in brackets)(Akademie Nieuws July 2011 p5)¹⁶⁴

But Freudenthal and his followers did claim that ‘realistic mathematics’ was scientifically warranted and based upon evidence. Thus professor in mathematics Lenstra observes a fraud with respect to empirical research, but doesn’t do anything about it. He doesn’t call for a repeal and annulment of earlier “research” that claimed empirical relevance but without such base. He is quite happy that his fellow abstract thinking mathematician Freudenthal invented a theory and let others suffer the consequences. Lenstra is another abstract thinking mathematician who now thinks that he has solved a problem and then lets others suffer the consequence – like the international community that still considers Freudenthal’s work “research”.

Lenstra’s solution to the arithmetic problem in Dutch primary education is not to retrain the 150,000 elementary school teachers, but shift the problem to the 4,000 math teachers in secondary education. The formula is that “arithmetic skills must be maintained” in the highschool curriculum. Lenstra suggests that it must be tested, but doesn’t quite specify how. The Dutch state secretary on education, Sander Dekker,¹⁶⁵ wants mandatory arithmetic tests for highschool graduation. If you cannot calculate with pen and paper then you can’t get your highschool diploma. Lenstra thinks that this is too strict (see here¹⁶⁶) but doesn’t provide a practical alternative how to test whether arithmetic skills have been “maintained”. The state secretary apparently is quite happy that he doesn’t have to retrain the 150,000 elementary school teachers, many of whom are likely to fail too, and that it suffices to increase the burden for the 4,000 secondary school teachers, and of course the burden for the kids who turn 16 or 18 and discover that the educational system has given them a raw deal. (It is a bit too easy to blame them that they should have worked harder.)

One might say that Lenstra’s 2009 KNAW Report and recent June 30 2014 KNAW conference presentation (my report in Dutch)¹⁶⁷ aren’t quite research themselves but rather evaluations on educational policy. It may well be that Lenstra’s texts here don’t register under scientific integrity in a strict legal sense, even though Parliament regards it as scientifically warranted. In another respect, Lenstra’s case is just an example, and it is a collective problem that abstract thinking mathematicians expound about empirical issues that they haven’t studied. Hence, my letter to KNAW-LOWI suggests a general exploration into the issue, so that the scientific community grows aware of the issue. Hopefully the specific issues on Freudenthal and his Institute are taken along, as explained here¹⁶⁸.

PM 1. Above mentioning of Abstraction vs Empirics might cause the idea that those would be opposite, but these are rather separate axes. We might score the different professional groups on the study hours in each category, with mean and dispersion. I mention two example individuals – Andrew Gelman and Richard Gill – for lack of an accepted term like “empirical mathematical statistics”. Since teachers teach they aren’t in research like other professions.

PM 2. This weblog concentrates on failure on integrity within Dutch economic science, with the case at the Dutch Central Planning Bureau (CPB) concerning economic co-ordination and the example of unemployment. That censored analysis is relevant for the

¹⁶⁴ https://www.know.nl/shared/resources/actueel/Akademienieuws/pdf/AN110_Rekenonderwijs.pdf

¹⁶⁵ http://nl.wikipedia.org/wiki/Sander_Dekker

¹⁶⁶ <http://wisforum.nl/hoorzitting/lenstra.pdf>

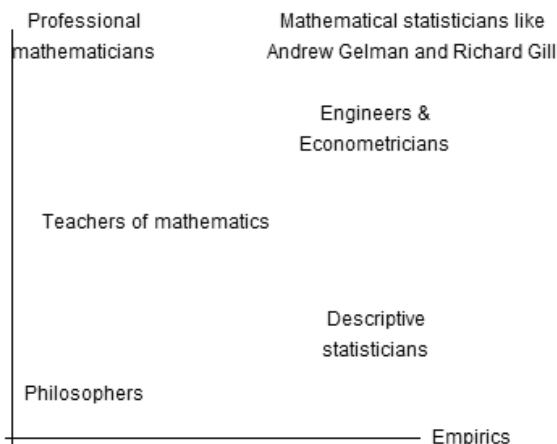
¹⁶⁷ <http://thomascool.eu/Papers/Math/2014-07-02-KNAW-Rekenen-deugt-niet.html>

¹⁶⁸ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

current crisis in the European Union, and for economic recovery in the United States, and for economic policy in the “emerging markets” too. My advice is to boycott Holland till the censorship of economic science here is resolved. KNAW-LOWI cannot officially tackle the case since its mandate concerns universities while the CPB falls directly under the national government.

A model of Abstraction vs Empirics

Abstraction = Mathematics



Example scores in the Abstraction vs Empirics space (might be tested)

But now there is a breach in integrity in research in math education as well. I have two academic degrees, one in econometrics and one in teaching mathematics, and it is disappointing to observe that my degrees open up to vista's of non-integrity. It might soon become a personal thing. But, as Art Buchwald would advise us: while there is a whole country to blame we might as well take a look at the facts. And boycott that country till they get their act together.

The world chooses to use English rather than Dutch or Latin, and hence we fall in the Dutch language sink again.¹⁶⁹ As my letter (above) is in Dutch, we might need to look to whom in Belgium, South Africa or Suriname still understands the gibberish. I presume that the people in New York (“New Amsterdam”, if they only knew) would need a certified translator. There might be some expats living in Holland who have learned some Dutch but I am afraid that some issues are getting complex, and then even many Dutch people would not be able to follow things. Foreigners would have even more difficulty in understanding local conditions. Hence I am quite hesitant to translate that letter.

For comparison on KNAW-LOWI, we may look at the US bureau for research in education, IES,¹⁷⁰ and the office of research integrity ORI of public health,¹⁷¹ and then also the ethical codes of AMS¹⁷² and MAA.¹⁷³ There are a lot of ethics to look into.

But let us not forget about education itself, and thus let me also alert you to this issue of CF with *Forty Years of Radical Constructivism in Educational Research*¹⁷⁴ and hope that they put students before method.

¹⁶⁹ <https://boycottholland.wordpress.com/2012/03/10/spinoza-and-the-crazy-centuries/>

¹⁷⁰ <http://ies.ed.gov/aboutus/>

¹⁷¹ <http://ori.hhs.gov/about-ori>

¹⁷² <http://www.ams.org/about-us/governance/policy-statements/sec-ethics>

¹⁷³ <http://www.maa.org/about-maa/policies-and-procedures/code-of-ethics-and-whistle-blower-protection-policy>

¹⁷⁴ <http://www.univie.ac.at/constructivism/journal//9/3>

A disciplinary board for mathematicians

2013-03-05 ¹⁷⁵

You can read this best while listening to the beautiful Eleftheria Arvanitaki and her rousing *Metrisa* on YouTube. ¹⁷⁶ I don't know what she actually sings about, and I don't want to know, but you can sense the thunder clouds forming on the horizon, with already some flashes of lightning.

The Italian election result feels like that too. A deadlock, commentators say. Europe is in crisis again, they hold. Perhaps it is a blessing in disguise. Belgium needed 18 months to create the Elio di Rupo government, and many thought that the country actually did well and got a much needed period of rest.

There is discussion about a grand coalition of leftist Bersani and rightist Berlusconi, or perhaps a minority government but supported by Grillo. Grillo wants to judge proposals on their merits separately and refuses to bargain about a full programme. In these political analyses the habit of thinking in majority governments wreaks havoc again. According to this view you team up to kick out a minority. Why would it not be possible to have an inclusive government, in which (most of) all parties partake ? It is a good idea indeed to judge proposals on their merits, to keep out pork barrel, but still with the intention that all have to live in the same country.

We see an example in this duet ¹⁷⁷ with Arvanitaki. The somewhat aged beauty who sings *Na Na* and who keeps breaking her earthenware bells is Haris Alexiou, who has produced wonderful albums and who performed in great concerts. Listen to her stately singing, the vivacious *apopse* ¹⁷⁸ with images from her younger years, and the touching *oles*, ¹⁷⁹ a classic. Or *Ximeroni* ! ¹⁸⁰ (Unfortunately with bad sound quality.)

I admit, illustrating Italian election results by Greek singers may come across as farfetched, but the association actually is rather sound, because co-operation is also a form of art. Many people regard democracy and elections as sport, and they only notice the winner. In reality, those winners and their supporters may be the barbarians, lost to culture and civilisation.

Some suggest to break the deadlock in Italy with new elections. In that case we might see what I have been advising for some years now: (1) governments that mirror parliament, (2) annual elections. This gives voters more power and still forces politicians to co-operate. Two other crucial advices: (3) select the prime minister with a Borda Fixed Point method, so that he or she has broad support and still can function in impartial manner above the parties (see this application to Holland), ¹⁸¹ (4) create an Economic Supreme Court that watches over the quality of information. These four elements improve the responsiveness to popular sentiment without turning into populism, and they increase the quality without turning into technocracy.

Notice that there is a fundamental problem here. Election methods normally are a disaster. In the US election between Bush, Gore and Nader the winner was Bush while Gore would have beaten each of the others in pairwise comparisons. In the French election between Chirac, Jospin and LePen the winner was Chirac, while Jospin

¹⁷⁵ <https://boycottholland.wordpress.com/2013/03/05/a-disciplinary-board-for-mathematicians/>

¹⁷⁶ <http://www.youtube.com/watch?feature=endscreen&v=QikmJP6sRQM&NR=1>

¹⁷⁷ <http://www.youtube.com/watch?v=FiLU7hMxj-k>

¹⁷⁸ http://www.youtube.com/watch?v=W_baXNQylqo&NR=1&feature=endscreen

¹⁷⁹ <http://www.youtube.com/watch?v=aLyXnESdXUc&NR=1&feature=endscreen>

¹⁸⁰ <http://www.youtube.com/watch?v=PuHliLPeFtw&feature=endscreen&NR=1>

¹⁸¹ <http://mpr.ub.uni-muenchen.de/44158/>

would have beaten each of the others in pairwise comparisons. Lawyers who write electoral laws tend not to understand much of mathematics, and then ask advice from mathematicians, who however create math from thin air and apply it to reality without understanding reality. Democracy disappears in the ravine between alpha's and beta's, the Two Cultures of C.P. Snow.¹⁸²

Italy has developed a complex system to allocate seats with the intention to enhance stability. That system now seems to enhance instability. It might be that it works out okay, as we hope above, but that would be by chance or wisdom, and the electoral system remains a disaster.

Of the many people who have been sleeping, a great responsibility falls on the politicians who voted this system into action. There is also a responsibility for the mathematicians who have been advising in the background. The fundamental problem is they can help to design systems, but run away from criticism, do not acknowledge error, and thus do not learn from mistakes. With their structurally erroneous advices these mathematicians destroy huge democracies.

We need a disciplinary board for mathematicians. When a medical doctor gives a wrong advice then there is such a board. A mathematician who doesn't study reality but still advises that abstract notions apply to reality, is condemnable in the same manner.

In a short Dutch article *Pas op met wiskunde over verkiezingen*¹⁸³ I explain the issue at a level for highschool students. Its appendix also contains a list of some 10 mathematicians who run away from criticism on their work on voting and democracy. In 1990 I observed that serious errors were being made, and the list gives that experience of denial since then. The list contains only mathematicians who are supposed to have an ethic of 'definition, theorem, proof' and who sin against that, even when the error is pointed out to them. It is no use to make a list of economists and political scientists who repeat the errors of the mathematicians, since that list would be much longer, and they would tend to refer to the mathematicians anyway (as if that would be proper).

The story turns into horror. I do not know whether I should refer to the dancing and waving of Eleni Bitali and her song about her life (*zoi mou*).¹⁸⁴ Beware: first it seems as if she is the blond lady but later the camera switches position and it appears that she is the lady with red hair. I offered above Dutch article to the journal of the Dutch Association for Mathematics *Nieuw Archief voor Wiskunde* (NAW).¹⁸⁵ One would think that the editors would be delighted with a short exposition of the major errors by mathematicians on voting theory and democracy. At that, a discussion that high school students should be able to understand, and that reviews which mathematician better corrects which misunderstanding. One would expect that the editors would desire to advance better mathematics. But no. Editor Barry Koren of the University of Leiden answers that he has studied the short article, fails to understand it, doesn't specify what passage he doesn't understand, rejects the paper and closes the discussion, in one grand sweep. I have included his name on the list of failing mathematicians because of this event, though as far as I know he hasn't written on voting theory. But the horror is that this concerns criticism on the math profession and that a journal blocks that criticism.

Perhaps professor Koren of Leiden didn't understand that the article was targetted at a level of exposition for highschool students, though it was explained to him. Perhaps he mistook the easy language with sloppy thinking. Perhaps he wanted to see complex mathematics though he could have found these in the references. We can imagine

¹⁸² http://en.wikipedia.org/wiki/The_Two_Cultures

¹⁸³ <http://thomascool.eu/Thomas/Nederlands/Wetenschap/Artikelen/2013-02-14-PasOpMetWiskundeOverVerkiezingen.html>

¹⁸⁴ <http://www.youtube.com/watch?v=l6UhFZ9h838>

¹⁸⁵ <http://www.nieuwarchief.nl/>

various misunderstandings. The fundamental point is that he presents a closed mind. An econometrician is not allowed to criticize mathematicians when they don't study reality but still give advice on that.

Do the mathematicians fail only on democracy and election methods ? No, they do so too in the education of mathematics, when they have been trained for abstract thought and suddenly encounter real life pupils. They do so too when they are 'rocket scientists' and develop financial products that do not account for real risks. They do so too in the study of logic when they exclude nonsense while that is the most nonsensical thing to do. I only mention areas that I have studied myself and where I have established this. Perhaps other people have other examples.

My advice for a disciplinary board for mathematicians thus is dead serious.

We end with Eleftheria Arvanitaki and a sirtaki ¹⁸⁶ in the studio. Listen especially from minute 37 onwards, when the guests have unpacked their presents, and Eleftheria enchants all hearts, with all eyes becoming watery and proud men holding on to their sigaret, and with Haris Alexiou in full rapture.

When Greek singers and their musicians would travel over Europe and would teach us to sing and dance then they have another export product with great potential, alongside with those earlier ideas about democracy and mathematics.

¹⁸⁶ <http://www.youtube.com/watch?v=ApAg5qMddNY>

Robbert Dijkgraaf as Darth Vader in Princeton

2012-03-08 ¹⁸⁷

Darth Vader is iconic and doesn't need explanation. ¹⁸⁸

People feel at ease though when the obvious is being explained so let me make you feel at ease. The universe is filled with a Force that Jedi knights apply to the Good. Anakin is very sensitive to the Force and becomes the chief Jedi, but he appears to have essential weaknesses and the Force corrupts him. He turns into Darth Vader and becomes the chief villain of the evil emperor.

Robbert Dijkgraaf may become iconic if he really turns into Darth Vader. ¹⁸⁹ He is departing as President of the Dutch Academy of Sciences (KNAW) ¹⁹⁰ and becomes director of the Institute of Advanced Study (IAS) in Princeton. ¹⁹¹

When Albert Einstein fled from Nazi Germany he came to IAS.

Also Kurt Gödel settled down there, and walked with Einstein.

I have my doubts about these two gentlemen. My book *Conquest of the Plane* (COTP) section 14.2 discusses that when space has been defined then we lose freedom to redefine it again. The human definition of space is Euclidean. We imagine non-Euclidean geometry (say on a sphere) within Euclidean definitions. Einstein's mathematical manipulation of space is actually a way to deal with measurement problems, and it is not warranted that this is a good way. My book *A Logic of Exceptions* (ALOE) chapter 9 shows that reasonable assumptions cause the Gödelian to collapse to the Liar paradox that can be solved with three-valued logic, so that Gödel's conclusions only apply to artificially weak systems.

These reservations about these two gentlemen must be seen in perspective. We might criticize Aristotle that he hasn't developed differential calculus so that his books have less relevance today. But this is silly. Aristotle is one of the great geniuses of mankind so we must appreciate his results within the framework of his time. Perhaps the same holds about Einstein. He died only recently so we may need some more centuries to get a proper perspective. At least I do since I don't claim to understand modern physics at this moment. About Gödel there is more room for doubt since it is at least as likely that he was a deluded mathematician. But of course I may be wrong. Anyhow, these two gentlemen have greatly contributed to the high standing of IAS in academic culture.

We can identify the Force with Science. Thus scientists can use Science in the service of the Good. Subsequently there is Robbert Dijkgraaf (Anakin) who is very sensitive to Science and he got elected to be President of KNAW (becoming the chief Jedi).

I wrote him a message about the censorship of science by the Dutch Central Planning Bureau (CPB). Dijkgraaf did not respond and didn't do anything. Legally, the CPB does not reside under KNAW but under the Ministry of Economic Affairs. Nevertheless, KNAW has a role in Dutch society to watch over the integrity of science and it is not unreasonable to expect that Dijkgraaf should step in, and urge the government to deal with the case of integrity of science at CPB. None of this.

Thus in my experience Dijkgraaf has failed as a scientist and President of KNAW. Integrity hasn't been defended where it mattered most. Dijkgraaf opted for his own

¹⁸⁷ <https://boycottholland.wordpress.com/2012/03/08/robbert-dijkgraaf-as-darth-vader/>

¹⁸⁸ Just to be sure: http://en.wikipedia.org/wiki/Darth_Vader

¹⁸⁹ http://en.wikipedia.org/wiki/Robbert_Dijkgraaf

¹⁹⁰ <http://www.knaw.nl/smartsite.dws?id=25792&lang=ENG>

¹⁹¹ <http://www.ias.edu/>

agenda. He has the essential weaknesses of vanity and fear (for fear). He paraded all over Dutch television promoting science but it was a farce because he allowed the government to destroy the integrity.

The IAS loved the show and loved to have Dijkgraaf. Has it turned into the Evil Empire ? The IAS exists by cherry-picking apparent winners and then advertising how good they all are. It is not clear whether this breeds a critical scientific attitude. The Institute states: "For 2012-2013 the theme will be Economics and Politics".¹⁹² They would need my book DRGTPE on the failure of the Trias Politica and the necessity of an Economic Supreme Court.¹⁹³ But it is unlikely that they use it because of the censorship in Holland. Will IAS criticize Dijkgraaf's performance in Holland ? IAS will put out corrupted science.

PS. The IAS *Institute Letter* Fall 2011¹⁹⁴ announces the appointment of Dijkgraaf and also discusses the 'continuum hypothesis'. Trained logicians have brainwashed themselves into irrationality, and are like a sect worthy of antropological research. See also my papers *Contra Cantor Pro Occam*¹⁹⁵ and *Neoclassical Mathematics for the Schools*¹⁹⁶ - both superseded by FMNAI (2015).

PPS. I wrote the editor of the IAS *Institute Letter* about all this but haven't gotten an answer.

¹⁹² <http://www.sss.ias.edu/>

¹⁹³ <http://thomascool.eu/Papers/Drgtpe/Index.html>

¹⁹⁴ <http://www.ias.edu/files/pdfs/letter-2011-fall.pdf>

¹⁹⁵ <http://thomascool.eu/Papers/ALOE/2011-08-07-ContraCantorProOccam.pdf>

¹⁹⁶ <http://thomascool.eu/Papers/Math/2011-09-06-NeoclassicalMathematics.pdf>

Pierre van Hiele and Stellan Ohlsson

2015-09-03 ¹⁹⁷

Mathematics education research (MER) not only looks at the requirements of mathematics and the didactics developed in the field itself, but also at psychology on cognition, learning and teaching in general, at pedagogy on the development of pupils and students, and at other subjects, such as physics or economics for cases when mathematics is applied, or general philosophy indeed. The former weblog text (p133) said something about neuro-psychology. Today we have a look at cognitive psychology.

Stellan Ohlsson: Deep learning

Stellan Ohlsson ¹⁹⁸ (2011) *Deep Learning: How the Mind Overrides Experience* ¹⁹⁹ may be relevant for mathematics education. One teaching method is to get students to think about a problem until the penny drops. For this, Ohlsson discusses a bit more than the distinction between old and new experience:

“(…) the human mind also possesses the ability to override experience and adapt to changing circumstances. People do more than adapt; they instigate change and create novelty.” (cover text)

“If prior experience is a seriously fallible guide, learning cannot consist solely or even primarily of accumulating experiences, finding regularities therein and projecting those regularities onto the future. To successfully deal with thoroughgoing change, human beings need the ability to override the imperatives of experience and consider actions other than those suggested by the projection of that experience onto the situation at hand. Given the turbulent character of reality, the evolutionary strategy of relying primarily on learned rather than innate behaviors drove the human species to evolve cognitive mechanisms that override prior experience. This is the main theme of this book, so it deserves a label and an explicit statement:

The Deep Learning Hypothesis

In the course of shifting the basis for action from innate structures to acquired knowledge and skills, human beings evolved cognitive processes and mechanisms that enable them to suppress their experience and override its imperatives for action.” (page 21)

(picture of book cover)

Stellan Ohlsson's book (2011) (Source: CUP)

Definition & Reality methodology

The induction question is how one can know whether all swans are white. Even a statistical statement runs into the problem that the error is unknown. Skepticism that one cannot know anything is too simple. Economists have the question how one can make a certain general statement about the relation between taxation and unemployment.

My book DRGTPE ²⁰⁰ (2000, 2005, 2011) (PDF online) (though dating from 1990, see the background papers from 1992) proposes the *Definition & Reality methodology*. (1) The

¹⁹⁷ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson/>

¹⁹⁸ <http://psch.uic.edu/psychology/people/faculty/stellan-ohlsson>

¹⁹⁹ <http://www.cambridge.org/us/academic/subjects/psychology/cognition/deep-learning-how-mind-overrides-experience>

²⁰⁰ <http://thomascool.eu/Papers/Drgtpe/Index.html>

model contains definitions that provide for certainty. Best would be logical tautologies. Lack of contrary evidence allows room for other definitions. (2) When one meets a black “swan” then it is no swan. (3) It is always possible to choose a new model. When there are so many black “swans” that it becomes interesting to do something with them, then one can define “swan2”, and proceed from there. Another example is that in one case you must prove the Pythagorean Theorem and in the other case you adopt it as a definition for the distance metric that gives you Euclidean space. The methodology allows for certainty in knowledge but of course cannot prevent surprises in empirical application or future new definitions. The methodology allows DRGTPE to present a certain analysis about a particular scheme in taxation – the tax void – that causes needless unemployment all over the OECD countries.

Karl Popper (1902-1994)²⁰¹ was trained as a psychologist, and there met with the falsification approach by Otto Selz (1881-1943).²⁰² Popper turned this into a general philosophy of science. (Perhaps Selz already thought in that direction though.) The Definition & Reality methodology is a small amendment to falsificationalism. Namely, definitions are always true. Only their relevance for a particular application is falsifiably. A criterion for a scientific theory is that it can be falsified, but for definitions the strategy is to find general applicability and reduce the risk of falsification. In below table, Pierre van Hiele presented his theory of levels of insight as a general theory of epistemology, but it is useful to highlight his original application to mathematics education, with the special property of formal proof. Because of this concept of proof, mathematics may have a higher level of insight / abstraction overall. Both mathematics and philosophy also better take mathematics education research as their natural empirical application, to avoid the risk of getting lost in abstraction.

Addendum September 7: The above assumes sensible definitions. Definitions might be logically nonsensical, see ALOE or FMNAI. When a sensible definition doesn't apply to a particular situation, then we say that it doesn't apply, rather than that it would be *untrue* or *false*. An example is an econometric model that consists of definitions and behavioural equations. A definition that has no relevance for the topic of discussion is not included in that particular model, but may be of use in another model.

<i>(Un-) certainty</i>	<i>Definitions</i>	<i>Constants</i>	<i>Contingent</i>
<i>Mathematics</i>	Euclidean space	$\Theta = 2\pi$?
<i>Physics</i>	Conservation of energy	Speed of light	Local gravity on Earth
<i>Economics</i>	Savings are income minus consumption	Institutional (e.g. annual tax code)	Behavioural equations
<i>Mathematics education</i>	Van Hiele levels of insight	Institutional	Student variety

To my great satisfaction, Ohlsson (2011:234) adopts basically the same approach.

“The hypothetical process that supposedly transforms particulars into abstractions is called induction and it is often claimed to operate by extracting commonalities across multiple particulars. If the first three swans you ever see are white, the idea *swans are white* is likely to come to mind. However, the notion of induction is riddled with problems. How are experiences grouped for

²⁰¹ https://en.wikipedia.org/wiki/Karl_Popper

²⁰² https://en.wikipedia.org/wiki/Otto_Selz

the purpose of induction? That is, how does the brain know which experiences are instances of some abstraction X , before that abstraction has been learned? How many instances are needed? Which features are to be extracted? How are abstractions with no instances in human experience such as the *infinite*, the *future* and *perfect justice* acquired?"

Definition of abstraction

There is an issue w.r.t. the definition of abstraction though. Compare:

- My definition of abstraction is *leaving out aspects*, see here ²⁰³ on this weblog, and see FMNAI. ²⁰⁴ My suggestion is that thought itself consist of abstractions. Abstraction depends upon experience since experience feeds brain and mind, but abstraction does not depend upon *repeated* experience.
- Ohlsson (2011:16) takes it as identical to induction, which explains the emphasis upon experience in his title, rather taken as *repetition*: "Memories of individual events are not very useful in themselves, but, according to the received view, they form the raw material for further learning. By extracting the commonalities across a set of related episodic memories, we can identify the underlying regularity, a process variously referred to as *abstraction*, *generalization* or *induction*." For Ohlsson, thoughts do not consist of abstractions, but of *representations* (models): "In the case of human cognition – or the intellect, as it would have been called in the 19th century – the relevant stuff consists of *representations*. Cognitive *functions* like seeing, remembering, thinking and deciding are implemented by *processes* that create, utilize and revise representations." and "Representations are structures that refer to something (other than themselves)." (page 29)

Ohlsson has *abstraction* \Leftrightarrow *induction* (*commonality*). For me it is dubious whether induction really exists. The two pathways are too different to use equivalence. (i) Comparing A and B , one must first abstract from A and then abstract from B , before one may decide whether those abstractions are the same, and before one can even say that A and B share a commonality. (ii) An abstract idea like a circle might cause an "inductive" statement that all future empirical circles will tend to be round, but this isn't really what is meant by "induction" – which is defined as the "inference" from past swans to future swans.

For me, an abstraction can be a model too, and thus would fit Ohlsson's term *representation*, but the fact that he chooses *abstraction* \Leftrightarrow *induction* rather than *abstraction* \Leftrightarrow *representation* causes conceptual problems. Ohlsson's definition of abstraction seems to hinder his understanding of the difference between *concrete* versus *abstract* as used in mathematics education research (MER).

Concrete versus abstract

Indeed, Ohlsson suggests an inversion of how people arrive at *insight*:

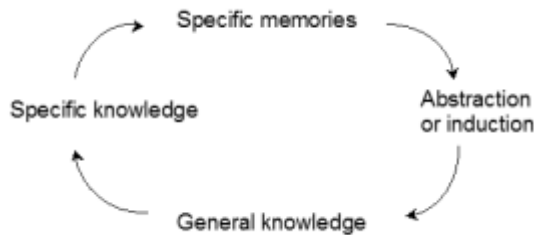
"The second contribution of the constraint-based theory is the principle that practical knowledge starts out general and becomes more specific in the course of learning. There is a long-standing tradition, with roots in the beginnings of Western philosophy, of viewing learning as moving in the opposite direction, from particulars to abstractions. [fnt 38 e.g. to Piaget] Particulars are given in perception while abstractions are human constructions, or so the ancient story goes." (p234)

²⁰³ <https://boycottholland.wordpress.com/2015/05/23/abstraction-vs-eugene-wigner-edward-frenkel/>

²⁰⁴ <http://thomascool.eu/Papers/FMNAI/Index.html>

“The fundamental principle behind these and many other cognitive theories is that knowledge moves from concrete and specific to abstract and general in the course of learning.” (Ohlsson 2011:434 that states fnt 38)

If I understand this correctly, and combine this with the earlier argument that general knowledge is based upon induction from specific memories, then we get the following diagram. Ohlsson’s theory seems inconsistent, since the specific memories must derive from specific knowledge but also presume those. Perhaps a foetus starts with a specific memory without knowledge, and then a time loop starts with cumulation over time, like the chicken-egg problem. But this doesn’t seem to be the intention.



Trying to understand Ohlsson’s theory of knowledge

There is this statement on page 31 that I find confusing since now abstractions [inductions ?] depend upon representations, while earlier we had them derived from various memories.

“The power of cognition is greatly increased by our ability to form abstractions. Mathematical concepts like the *square root of 2* and a *four-dimensional sphere* are not things we stumble on during a mountain hike. They do not exist except in our representations of them. The same is true of moral concepts like *justice* and *fairness*, as well as many less moral ones like *fraud* and *greed*. Without representation, we could not think with abstractions of any kind, because there is no other way for abstract entities to be available for reflection except via our representations of them. [fnt 18]”

Fnt 18 on page 402: “Although abstractions have interested philosophers for a long time, there is no widely accepted theory of exactly how abstractions are represented. The most developed candidate is schema theory. (...)”

My suggestion to Ohlsson is to adopt my terminology, so that thought, abstraction and representation cover the same notion. Leave induction to the philosophers, and look at statistics for empirical methods. Then eliminate representation as a superfluous word (except for representative democracy).

That said, we still must establish the process from concrete to abstract knowledge. This might be an issue of terminology too. There are some methodological principles involved however.

Wilbrink on Ohlsson

Dutch psychologist Ben Wilbrink ²⁰⁵ alerted me to Ohlsson’s book – and I thank him for that. My own recent book *A child wants nice and no mean numbers* ²⁰⁶ (CWNN) (PDF online) contains a reference to Wilbrink’s critical discussion of arithmetic in Dutch primary

²⁰⁵ <http://benwilbrink.nl/indexen.htm>

²⁰⁶ <http://thomascool.eu/Papers/NiceNumbers/Index.html>

schools. Holland suffers under the regime of “realistic mathematics education” (RME) that originates from the Freudenthal “Head in the Clouds Realistic Mathematics” Institute (FHCRMI) in Utrecht. This FHCRMI is influential around the world, and the world should be warned about its dismal practices and results. Here is my observation ²⁰⁷ that Freudenthal’s approach is a fraud.

Referring to Ohlsson, Wilbrink suggests ²⁰⁸ that the “level theory by Piaget, and then include the levels by Van Hiele and Freudenthal too” (my translation) are outdated and shown wrong. This, however, is too fast. Ohlsson indeed refers to Piaget (stated fnt 38) but Van Hiele and Freudenthal are missing. It may well be that Ohlsson missed the important insight by Van Hiele. ²⁰⁹ It may explain why Ohlsson is confused about the directions between concrete and abstract.

A key difference between Van Hiele and Freudenthal

CWNN pages 101-106 discusses the main difference between Hans Freudenthal (1905-1990) and his Ph.D. student Pierre van Hiele (1909-2010). Freudenthal’s background was abstract mathematics. Van Hiele was interested from early on in education. He started from Piaget’s stages of development but rejected those. He discovered, though we may as well say defined, levels of insight, starting from the concrete to the higher abstract. Van Hiele presented this theory in his 1957 thesis – the year of Sputnik – as a general theory of knowledge, or epistemology.

Freudenthal accepted this as a thesis, but, mistook this as the difference between pure and applied mathematics. When Freudenthal noticed that his prowess in mathematics was declining, he offered himself the choice of proceeding his life with the history of mathematics or the education of mathematics. He chose the latter. Hence, he coined the phrase *realistic mathematics education* (RME), and elbowed Van Hiele out of the picture. As an abstract thinking mathematician, Freudenthal created an entire new reality, not caring about the empirical mindset and findings by Van Hiele. One should really read CWNN pages 101-106 (PDF online) for a closer discussion of this. Van Hiele’s theory on knowledge is hugely important, and one should be aware how it got snowed under.

A recent twist in the story is that David Tall (2013) rediscovered Van Hiele’s theory, but wrongly holds (see here ²¹⁰) that Tall himself found the general value while Van Hiele had the misconception that it only applied to geometry. In itself it is fine that Tall supports the general relevance of the theory of levels.

The core confusion by Ohlsson on concrete versus abstract

The words “concrete” and “abstract” must not be used as absolutely fixed in exact meaning. This seems to be the core confusion of Ohlsson w.r.t. this terminology.

When a child plays with wooden blocks we would call this concrete, but our definition of thought is that thinking consists of abstractions, whence the meanings of the two words become blurred. The higher abstract achievement of one level will be the concrete base for the next level. The level shift towards more insight consists of compacting earlier insights. What once was called “abstract” suddenly is called “concrete”. The statement “from concrete to abstract” indicates both the general idea and a particular level shift.

Van Hiele’s theory is essentially a logical framework. It is difficult to argue with logic:

²⁰⁷ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

²⁰⁸ <http://benwilbrink.nl/literature/ohlsson.htm>

²⁰⁹ He confirmed this in a subsequent email.

²¹⁰ <http://arxiv.org/abs/1408.1930>

1. A novice will not be able to prove laws or the theorems in abstract mathematics, even informally, and may even lack the notion of proof. Having achieved formal proof may be called the highest level.
2. A novice will not be able to identify properties and describe their relationships. This is clearly less complex than (1), but still more complex than (3). There is no way going from (3) to (1) without passing this level.
3. A novice best starts with what one knows. This is not applied mathematics, as Freudenthal fraudently suggested, but concerns the development of abstractions that are available at this level. Thus, use experience, grow aware of experience, use the dimensions of text, graph, number and symbol, and develop the thoughts about these.

Van Hiele mentioned five levels, e.g. with the distinction between informal and formal deduction, but this is oriented at mathematics, and above trident seems sufficient to establish the *generality* of this theory of knowledge. A key insight is that words have different meanings depending upon the level of insight. There are at least three different languages spoken here.

Three minor sources of confusion are

- Ohlsson's observation that one often goes from the general to the specific is correct. Children may be vague about the distinction between "a man" and "one man", but as grown up lawyers they will cherish it. This phenomenon is not an argument against the theory of levels. It is an argument about becoming precise. It is incorrect to hold that "one man" is more concrete and "a man" more abstract.
- There appears to exist a cultural difference between on one side Germans who tend to require the general concept (*All men are mortal*) before they can understand the particular (*Socrates is mortal*), and the English (or Anglo-Saxons who departed from Germany) who tend to understand only the particular and to deny the general. This cultural difference is not necessarily epistemological.
- Education concerns knowledge, skill and attitude. Ohlsson puts much emphasis on skill. Major phases then are arriving at a rough understanding and effectiveness, practicing, mastering and achieving efficiency. One can easily see this in football, but for mathematics there is the interplay with the knowledge and the levels of insight. Since Ohlsson lacks the levels of insight, his phases give only part of the issue.

Conclusion

I have looked only at parts of Ohlsson's book, in particular above sections that allow a bit more clarity on the relevance w.r.t. Van Hiele's theory of levels of insight. Perhaps I read more of Ohlsson's book later on, but this need not be soon. Please understand my predicament.

- In mathematics education research (MER) we obviously look at findings of cognitive psychology, but this field is large, and it is not the objective to become a cognitive psychologist oneself.
- When cognitive psychologists formulate theories that include mathematical abstraction, as Ohlsson does, let them please look at the general theory on knowledge by Pierre van Hiele, for this will make it more relevant for MER.
- Perhaps cognitive psychologists should blame themselves for overlooking the theory by Pierre van Hiele, but they also should blame Hans Freudenthal, and support my letter to IMU / ICMI ²¹¹ (see p53 above) asking to correct the issue.

²¹¹ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

They may work at universities that also have departments of mathematics and sections that deal with MER, and they can ask what happened.

- When there is criticism on the theory by Van Hiele, please look first at the available material. There are summary statements on the internet, but these are not enough. David Tall looked basically at one article and misread a sentence (and his misunderstanding still was inconsistent with the article). For some references on Van Hiele look here.²¹² (There is the Van Hiele page²¹³ by Ben Wilbrink, but, as said, Wilbrink doesn't understand it yet.)

²¹² <http://arxiv.org/abs/1408.1930>

²¹³ <http://benwilbrink.nl/literature/hiele.htm>

Pierre van Hiele and Ben Wilbrink

2015-09-05²¹⁴

To my surprise, today gives more on psychology. Since highschool I denote this as Ψ . I appreciate social Ψ (paper 1996)²¹⁵ but am not attracted to other flavours of Ψ .

Last week we looked at some (neuro-) Ψ on number sense,²¹⁶ and a few days ago at some cognitive Ψ .²¹⁷ Dutch readers may look at some comments²¹⁸ last year w.r.t. the work by Leiden Ψ metrist Marian Hickendorff who explains that she is no expert on math education but still presents research on it.

Today I will look at what Dutch Ψ ist and education researcher Ben Wilbrink²¹⁹ states about the work by math education researcher Pierre van Hiele (1909-2010). I already observed a few days ago that Wilbrink didn't understand Van Hiele's theory of levels of insight. Let me become more specific.

ME and MER are a mess, but Ψ maybe too

The overall context is that math education (ME) and its research (MER) are a mess. Mathematicians are trained for abstraction and cannot deal well with real existing pupils and the empirical science of MER.

When Ψ has criticism on this, it will be easy for them to be right.

Unfortunately, Ψ appears to suffer from its own handicap. Ψ people namely study Ψ . They do not study ME or MER. Ψ ists invent their own world full of Ψ theories *alpha* to *omega*, but it is not guaranteed that this really concerns ME and MER. We saw this in (neuro-) Ψ and in cognitive Ψ in above weblog texts. It appears also to hold for Wilbrink. Whether Ψ is a mess I cannot judge though, since I am no Ψ ist myself.

Ψ itself has theories about how people can be shortsighted. But we don't need such theory. A main element in the explanation is that Ψ ists tend to regard mathematicians as the experts in ME, while those are actually quite misguided. A mathematician's view on ME tends put the horse behind the carriage. Then Ψ comes around to advise ways to do this more efficiently.

When Pierre van Hiele criticises conventional MER, then Wilbrink comes to the fore to criticise Van Hiele:

1. for not knowing enough of Ψ ,
2. and for doing proposals that other mathematicians reject.

Welcome in the wonderful world of Kafka Ψ .

This has become an issue of research integrity

I have asked Ben Wilbrink to correct some misrepresentations. He refuses.

He might have excellent reasons for this. My problem is that he doesn't state them. I can only guess. One potential argument by Wilbrink is that he does Ψ . Perhaps he means to say that when I would get my third degree in Ψ too then I might better understand his misrepresentations. This is unconvincing. A misrepresentation remains a misrepresent-

²¹⁴ <https://boycottholland.wordpress.com/2015/09/05/pierre-van-hiele-and-ben-wilbrink/>

²¹⁵ <http://thomascool.eu/Papers/Institute/Institute.html>

²¹⁶ <https://boycottholland.wordpress.com/2015/08/29/research-on-number-sense-tends-to-be-invalid>

²¹⁷ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson>

²¹⁸ <http://thomascool.eu/Papers/Math/2014-07-02-KNAW-Rekenen-deugt-niet.html>

²¹⁹ <http://benwilbrink.nl/publicaties/cv.htm>

ation, whatever the amount of Ψ you put into it. Unless Wilbrink means to say that Ψ is misrepresentation by itself. Perhaps.

But: *Wilbrink's refusal to provide answers to some questions turns this into an issue in research integrity.*

Wilbrink (1944, now 70+) originally worked on the Ψ approach to test methodology (testing people rather than eggs). See for example the Item Response theory²²⁰ by Arpad Elo and Georg Rasch, also discussed in my book *Voting Theory for Democracy*.²²¹ The debate in Holland on dismal education in arithmetic causes Wilbrink to emphasize the (neglected) role of Ψ . He also tracks other aspects, e.g. his website lists my book *Elegance with Substance* (EWS) (2009),²²² but he makes his own selection. Perhaps he hasn't read EWS. At least he doesn't mention my advice to a parliamentary enquiry into mathematics education. All this is fine with me, and I appreciate much of Wilbrink's discussions.

However, now there is this issue on research integrity.

Let us look at the details. The basic evidence is given by Wilbrink's webpage (2012)²²³ on Pierre and Dina van Hiele-Geldof (retrieved today).

1. Having a hammer turns everything into a nail (empirics)

If you want to say something scientifically about mathematics education (ME), then you enter *mathematics education research* (MER).

- When you meet with criticism by people in MER that you overlook some known results, then check this.
- Ben Wilbrink overlooks some known results.
- But he refuses to check those, even when asked to.

In particular, he states that the Van Hiele *theory of levels of insight* would not be empirical.

But my books and weblog texts, also this recent one (page 69 above),²²⁴ explain that it is an empirical theory. I informed him about this. Wilbrink must check this, ask questions when he doesn't understand this, and give a counterargument if he does not agree. But he doesn't do that. What he does, is neglect MER, and simply state his view, and neglect this criticism. Thus:

- he misrepresents scientific results,
- he assumes a professional qualification that he doesn't have,
- and he misinforms his readership.

2. Having a hammer turns everything into a nail (Ohlsson)

Wilbrink (here,²²⁵ w.r.t. p233 fnt 38) adopts Ohlsson's inversion of the learning direction from concrete to abstract, and then rejects Van Hiele's theory. However, proper understanding of Van Hiele's theory shows that Ohlsson's inversion is empirically untenable.

²²⁰ https://en.wikipedia.org/wiki/Item_response_theory

²²¹ <http://thomascool.eu/Papers/VTFD/Index.html>

²²² <http://thomascool.eu/Papers/Math/Index.html>

²²³ <http://benwilbrink.nl/literature/hiele.htm>

²²⁴ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson/>

²²⁵ <http://benwilbrink.nl/literature/ohlsson.htm>

- Wilbrink doesn't react to the explanation ²²⁶ how Van Hiele's theory (how learning really works) shows Ohlsson's theory empirically untenable.
- As a scientist Wilbrink should give a counterargument, but he merely neglects it.

3. Having a hammer turns everything into a nail (Freudenthal)

A third case that Wilbrink (here, ²²⁷ w.r.t. p233 fnt 38 again) shows that he doesn't understand the subject he is writing about, is that he lumps Van Hiele and Freudenthal together, i.e. on the theory of levels. But their approaches are quite different. Van Hiele has *concrete versus abstract*, Freudenthal has *pure versus applied mathematics*. Freudenthal's conceptual error is not to see that you already must master mathematics before you can do applied mathematics. You will not master mathematics by applying it when you cannot apply it yet. *Guided reinvention* is a wonderful word, like *sim sala bim*.
²²⁸

It is a huge error by Wilbrink to not see this distinction. Wilbrink doesn't know enough about MER. This turns from sloppy science into an issue of research integrity when he does not respond to criticism on this.

Remarkably, Wilbrink (here, ²²⁹ on *Structure and Insight*) rightly concludes that Van Hiele is critical of Freudenthal and doesn't actually belong to that approach. Apparently, it doesn't really register. Wilbrink maintains two conflicting notions in his mind, and doesn't care. (See also points 10 and 14 below.)

4. Having a hammer turns everything into a nail (Kant)

Wilbrink looks at ME and MER from the angle of Ψ . This looks like a valuable contribution. He however appears to hold that only Ψ is valid, and MER would only be useful when it satisfies norms and results established by Ψ . This is scientifically unwarranted.

- There are cases in which Ψ missed insights from MER. See above. I have noted no Ψ ist making the observations that can be found in *Elegance with Substance*.
²³⁰
- The Van Hiele theory is a general theory in epistemology (see here ²³¹), and thus also Ψ must respect that. When Wilbrink doesn't do that, he should give an argument.

A conceivable argument by Wilbrink might be that Van Hiele did not publish a paper in a journal on philosophy (my notation Φ) so that the sons and daughters of Kant could have hailed it as a breakthrough in epistemology. The lack of this seal of approval might be construed as an argument that Ψ and Wilbrink would be justified to neglect it. This would be an invalid argument. When Wilbrink studies MER and Van Hiele's theory of levels, and reads about Van Hiele's claim of general epistemological relevance, then every academic worth his or her salt on scientific methodology, and especially Ψ ists, can recognise it for what it is: a breakthrough.

5. Having a hammer turns everything into a nail (testing validity)

Wilbrink's question whether there has been any testing on validity on Van Hiele's theory at first seems like a proper question from a Ψ ist, but neglects the epistemological status of the theory. He would require from physicists that they "test" the *law of conservation of*

²²⁶ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson/>

²²⁷ <http://benwilbrink.nl/literature/ohlsson.htm>

²²⁸ https://en.wikipedia.org/wiki/Harry_August_Jansen

²²⁹ <http://benwilbrink.nl/literature/hiele.htm>

²³⁰ <http://thomascool.eu/Papers/Math/Index.html>

²³¹ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson/>

energy, or from economists that they “test” that *savings are what remain from income after consumption*. This is quite silly, and only shows that Wilbrink did not get it. Perhaps his annoyance about Freudenthal caused him to attack Van Hiele as well ? Wilbrink should correct his misrepresentation, or provide a good reason why being silly is good Ψ .

6. Having a hammer makes you require that everyone is hammering

Wilbrink suggests that Pierre and Dina Van Hiele – Geldof performed “folk psychology”. This runs counter to the fact that Pierre studied Piaget, and explicitly rejected Piaget’s theory of stages. His 1957 thesis (almost 60 years ago) has three pages of references that include also other Ψ . Perhaps Wilbrink requires that they should have studied more of Ψ . That might be proper when the objective was to become a Ψ ist. But the objective was to do MER. Dina did the thesis with Langeveld,²³² a pedagogue, and Pierre with Freudenthal, mathematician and not known yet for the educational theories that he stole from Pierre (and distorted, but it remains stealing).

If the Ψ ists would succeed in presenting a general coherent and empirically corroborated theory, that every academic can master in say a year, then perhaps Ψ ists might complain that this is being neglected. Now that Ψ ists however create a wealth of different approaches, then researchers in MER are justified in selecting what is relevant for their subject, and proceed with the subject.

Wilbrink’s suggestion on “folk psychology” is disrespectful and slanderous.

7. Having a hammer makes you look for nails at low tide (pettifoggery)

Wilbrink reports that Dina van Geldof mentions only the acquisition of insight and does not refer to the relevance of geometry for a later career in society. Perhaps she doesn’t. Her topic of study was acquisition of insight. Perhaps Wilbrink only makes a factual observation. What is the relevance of this ? It is a comment like: “Dollar bills don’t state that people also use them in Mexico.” Since Wilbrink reports this in the context of above disrespectful “folk psychology”, the comment only serves to downgrade the competence of Dina van Geldof, and thus is slanderous. As if she would not understand it, when Pierre explained to her that his theory of levels had general epistemological value.

8. Having a hammer makes you look for nails in 1957

Wilbrink imposes norms of modern study design and citation upon the work of the Van Hieles in 1957 (when Pierre was 48). The few references in Pierre’s “*Begrip en inzicht*” (translated as “*Structure and insight*”, 2nd book, not the thesis) cause Wilbrink to hold, in paraphrase,

“by not referring, Van Hiele reduces his comments to personal wisdoms, by which he inadvertently downgrades them.”

This is a serious misrepresentation, even though the statement is that Van Hiele’s texts were more than just personal wisdoms.

(a) It is true that Van Hiele isn’t the modern researcher who always refers and is explicit about framework and study design. What a surprise. The observation is correct that norms of presentation of results have changed. Perhaps authors in the USA 1957 already referred, but this need not have been the case in Europe. (See a discussion on this w.r.t. John Maynard Keynes.²³³)

(b) The suggestion as if Van Hiele should have referred is false however. In that period the number of researchers and size of literature were relatively small, and an author could

²³² https://nl.wikipedia.org/wiki/Martinus_J._Langeveld

²³³ <https://boycottholland.wordpress.com/2014/10/26/thomas-robert-malthus-visiting-maastricht>

assume that readers would know what one was writing about. Some found it also pedantic to include footnotes.

Thus: (i) The lack of footnotes does not in any way reduce Van Hiele's comments to "personal wisdoms". Wilbrink is lazy and if he is serious about the issue then he should reconstruct the general state of knowledge in that period. (ii) The comment must be rewritten in what is factually correct, and the insinuation must be removed.

9. Having a hammer makes you put nails in other people's mouths

Wilbrink refers to an issue on fractions.²³⁴ He quotes Van Hiele's suggestion to use tables of proportions, which has been adopted by the Freudenthal Institute, and quotes criticism by modern mathematicians Kaenders & Landsman that those tables block insight into algebra.

This is a misrepresentation.

This is an example of that a Ψ ist quotes mathematicians as authorities, and regards their misunderstanding as infallible evidence. A student of MER however would (hopefully) see that there is more to it.

The very quote by Van Hiele contains his suggestion to look at multiplication. Indeed, the book "*Begrip en Inzicht*" chapter 22 contains a proposal to abolish fractions, and to deal with that algebraically – what Kaenders & Landsman may not know about.

The true criticism is that the *Freudenthal Head in the Cloud Realistic Mathematics Institute* mishandled Van Hiele's work: (a) selected only an easy part, and (b) did not further develop Van Hiele's real approach.

A proposal how Van Hiele's real approach can be developed is *here*.²³⁵ I agree with Kaenders & Landsman to the extent that presenting only such tables is wrong, and that also the algebraic relation should be specified. The student then has the option to use either, and learn the shift.

Curiously, Wilbrink²³⁶ comments on this chapter 22 with some approval. Thus he should have seen that he provided a false link between Van Hiele on tables of proportion and the critique by Kaenders & Landsman.

10. Having a hammer makes you hate who refuses to be a nail

Wilbrink discusses *Structure and Insight* (not the thesis) *here*.²³⁷ He quotes Van Hiele from p. viii:

"Many original ideas can be found in this book. I came upon them in analyzing dubious theories of both psychologists and pedagogues. It is not difficult to unmask such theories: simply test them in practice. Often this is not done because of the prestige of the theory's proponents."

Wilbrink's judgement (my translation):

"The quoted opinion is incredibly arrogant, lousy, or how do you call such a thing. Van Hiele is mathematician, and makes the same error here as Freudenthal made in his whole later life: judging the development of psychological theory not in the context of psychology, but in the context of one's own common sense. This clearly gives gibberish. Thus I will continue reading Van Hiele with extraordinary suspicion."

²³⁴ <http://benwilbrink.nl/projecten/breken.htm>

²³⁵ <https://boycottholland.wordpress.com/2014/09/04/with-your-undivided-attention/>

²³⁶ <http://benwilbrink.nl/literature/hiele.htm>

²³⁷ <http://benwilbrink.nl/literature/hiele.htm>

My comments on Wilbrink:

- Van Hiele was a mathematician but also a *teacher*, with much attention for the empirics of education. This is quite in contrast with Freudenthal who lived by abstraction. (Freudenthal did not create a professorship in math education for Van Hiele, but took the task himself.)
- Van Hiele does precisely what Wilbrink requires: look at Ψ and look at empirics (in this case: *practice*). The only thing what happens is that Van Hiele then rejects Ψ , and this is what Wilbrink doesn't swallow. While Van Hiele does MER, Wilbrink redefines this as Ψ , and then sends Van Hiele to the gallows for not sticking to some Ψ paradigm.
- It is useful to mention that Van Hiele does the same thing in the preface of his thesis. He states that Ψ theories have been shown inadequate (his references are three pages) and that he will concentrate on the notion of insight as it is used in educational practice. He opposes insight to rote learning, and mentions the criterion of being able to deal with new situations that differ from the learning phase.
- It is incorrect of Wilbrink to distinguish only the categories of either Ψ or "one's own common sense" or "folk psychology". It is quite obvious why Van Hiele cannot find in books on Ψ what he is looking for and actually does: *He presents his epistemological theory of levels*. Those aren't in those books on Ψ . If Van Hiele would do what Wilbrink requires, then he cannot present his theory of levels, since Wilbrink's strict requirements would force him to keep on barking up the wrong tree. It beats me why Wilbrink doesn't see that.

11. Having a hammer turns your foot into a nail

Wilbrink also quotes from viii:

"Some psychologies lay much stress on the learning of facts. The learning of structures, however, is a superior goal. Facts very often become outmoded; they sink into oblivion because of their lack of coherence. In a structure facts have sense; if part of a structure is forgotten, the remaining part facilitates recall of the lost one. It is worth studying the way structures work because of their importance for the process of thinking. For this reason a considerable part of my book is devoted to structures."

Wilbrink's comment on this is (my translation):

"For me this is psychological gibberish, though I rather get what Van Hiele intends (...)"

By which it is established that Wilbrink understands gibberish and may call gibberish what he understands.

12. Having a hammer makes that you run in a loop of nails

Wilbrink's subsequent quote from *Structure and Insight*:

"In this book you will find a description of a theory of cognitive levels. I show you how levels of thinking demonstrate themselves, how they come into existence, how they are experienced by teachers and how by pupils. You will also see how we can take account of those levels in writing textbooks."

Wilbrink (my translation):

“You cannot simply do this. At least Van Hiele must show by experiment that intersubjective agreement can be reached about who when what level has demonstrated by operational achievements (because we cannot observe thoughts directly). (...) Indeed, at least for himself it is evident. Can this idea be transferred to others ? Undoubtedly, for other people have invited him to make this English translation of his earlier book. But that is not the point. The crucial point is: does his theory survive empirical testing?”

My comment: It is a repetition of the above, but it shows that Van Hiele’s repeated explanation about the epistemological relevance of his theory for educational practice continues, time and time again, to elude Wilbrink’s frame of mind.

Of course, statistical science already established before 1957 that the golden standard of experimental testing consists of the double blind randomized trial. Instead, Van Hiele developed his theory over the course of years as teacher in practice. Though he mentions didactic observations already from his time as a student in highschool. But we are back in a repetitive loop when we must observe that it is false to require statistics for Van Hiele’s purposes.

13. Having a hammer makes you avoid number 13 for fear that it might make you superstitious

(picture)²³⁸

Hermann von Helmholtz, on the law of conservation of energy (source: wikimedia commons)

14. Having a hammer makes you miss a real nail

Wilbrink (2012) refers to the MORE study²³⁹ of 1993 that defined *realistic mathematics education* (RME) as consisting of:

- Van Hiele’s theory of levels
- Freudenthal’s didactic phenomenology
- the principle of progressive mathematizing according to Wiskobas (JStor).²⁴⁰

It is actually nice that Van Hiele is mentioned in 1993, for at least since 2008 he isn’t mentioned in the *Freudenthal Head in the Clouds Realistic Mathematics Institute* wiki on RME²⁴¹ (retrieved today). His levels have been replaced by Adri Treffler’s concept of “vertical mathematization”. Wilbrink might be happy that he doesn’t have to criticise the levels at FHCRMI anymore. It is now a vague mist that eludes criticism.

Wilbrink’s criticism of *Freudenthal’s didactic phenomenology* and *Wiskobas* are on target. It is indeed rather shocking that policy makers and the world of mathematics teaching went along with the nonsense and ideology. The only explanation is that mathematicians made a chaos with their *New Math*.²⁴² If Pierre van Hiele had been treated in scientific decent fashion, his approach would have won, but Freudenthal was in a position to prevent that.

Wilbrink apparently thinks that Van Hiele belongs to the Freudenthal group, even though he observes elsewhere that Van Hiele rejects this. *Wilbrink assumes both options, and his mind is in chaos.*

²³⁸ https://en.wikipedia.org/wiki/Conservation_of_energy

²³⁹ <http://benwilbrink.nl/projecten/more.htm>

²⁴⁰ <http://www.jstor.org/stable/3482879?seq=1>

²⁴¹ http://www.fisme.science.uu.nl/en/wiki/index.php/Realistic_Mathematics_Education

²⁴² https://en.wikipedia.org/wiki/New_Math

Wilbrink doesn't see that the Freudenthal clique only mentions Van Hiele to piggyback on his success, to manoeuvre him out, and later create some matching phrases so that Van Hiele doesn't have to be mentioned anymore.

The following is a repetition of point 5, but it can be found on this particular page & section by Wilbrink, and may deserve a comment too. Namely, regarding Van Hiele as a pillar of *realistic mathematics education*, Wilbrink states²⁴³ (my translation):

“Okay, I can infer that the theory of levels can be found in Van Hiele's thesis, but that thesis is of a conceptual nature, and it doesn't contain empirical research. Van Hiele doesn't deny the latter, see the passage on his pages 188-189; but that is really rather sensational: everyone parrots his theory of levels, without looking for empirical support. Every well-thinking person, who has read his Popper for example, can see that you can do just anything with that 'theory of levels': It is in the formulation by Van Hiele 1958 [article following the 1957 thesis ?] a theory that excludes almost nothing. I return to this extensively on the Van Hiele page.²⁴⁴”

My comments for completeness:

- Van Hiele's theory is as empirical as the *law of conservation of energy* or the economic principle that *savings are the remainder of income after consumption*. This is not pure mathematics but it applies to reality. Thus Van Hiele's theory is hugely empirical. See the former weblog text. (Page 69 above.)²⁴⁵
- Van Hiele's thesis p188-189 indeed mentions the subsequent relevance of statistical testing *to ground out details*. This is something else than testing on falsification. What Van Hiele states is not quite what Wilbrink suggests. The fact of the lack of statistical testing is correct. But Van Hiele does not subscribe to Wilbrink's criterion of “empiricism”.
- Van Hiele does not expect that there will be much statistical development of the levels. Therefor he judges that his theory will tend to be of more value for teachers in practical teaching.
- You can do with the *theory of levels* as much as with the *law of conservation of energy*. A bit, but a crucial bit. Who has read Popper will see that the idea of falsification must make an amendment on definitions.

Thus, if Wilbrink had had an open mind on epistemology, he could have nailed the FHCMI for producing nonsense and abusing the wonderful theory by Van Hiele. He missed.

But the key point is that he also misinforms his readership, and refuses to correct after he has been informed about it.

15. Having a hammer makes that only masochist nails like you

Wilbrink's discussion of Van Hiele's thesis chapter 1 (here,²⁴⁶ “*Wat is inzicht?*”) shows a lack of understanding about the difference between a theorem and a proof. Euclid turns in his grave.

Wilbrink makes a distinction between “mathematics and psychology of mathematics”, without explanation or definition, perhaps in the mood of writing for Ψ ists who will immediately smell the nest and cheer and be happy.

²⁴³ <http://benwilbrink.nl/projecten/more.htm>

²⁴⁴ http://benwilbrink.nl/projecten/van_Hiele.htm

²⁴⁵ <https://boycottholland.wordpress.com/2015/09/03/pierre-van-hiele-and-stellan-ohlsson/>

²⁴⁶ <http://benwilbrink.nl/literature/hiele.htm>

Wilbrink writes “Brrrrr” (check the r’s) when Van Hiele distinguishes insight based upon inference and insight based upon non-inference. Wilbrink does not explain whether his *Brrrrr* is based upon inference or non-inference.

Wilbrink fears that Van Hiele will base his didactic insight upon “reason” instead of “theory with empirical testing”. He does not explain what is against reasoning and teaching experience and reading in the literature, for developing a new theory. Perhaps Wilbrink thinks that true theories can only be found in books of Ψ ?

Wilbrink’s final judgement on Van Hiele’s thesis chapter 1 is that it is a “*tattle tale*”. It is a free world, and Wilbrink may think so and put this on his website. But if he wants to be seen as a scientist, then he should provide evidence. In this case, Van Hiele clearly stated that he found the Ψ theories useless, so that he returned to the notion of insight in educational practice. His discussion of what this means is clarifying. It links up with his theory of levels. Overall it makes sense. As an author he is free in the way how he presents his findings. He builds it up, from the concrete to the abstract. Wilbrink does not respect Van Hiele’s judgement, but provides no other argument than *Brrrrr*, or, the spraying with the label of Ψ , or, invoking the spell of the double blind randomized trial.

16. Having a hammer doesn’t make you a carpenter

Wilbrink (2012) ²⁴⁷ doesn’t comment on Van Hiele’s thesis’s final chapter XVIII about the relevance of the *theory of levels* for epistemology. An ostrich keeps its head in the sand, where it is warm and dark, like in the womb of its egg.

Conclusion

Originally, I saw some of Ben Wilbrink’s texts on Van Hiele before, and appreciated them for the discussion and references, since there is hardly anyone else in Holland who pays attention to Van Hiele. However, Wilbrink’s reaction to Ohlsson, to the effect that Van Hiele would be wrong about the learning direction of *concrete to abstract*, caused me to make this evaluation above.

Wilbrink maltreats Van Hiele’s work. Wilbrink doesn’t know enough about mathematics education research (MER) to be able to write about it adequately. He misinforms the public.

I have asked Wilbrink to make adequate corrections, or otherwise specify his (reply) arguments so that I could look into those. He refuses either. This constitutes a breach in the integrity of science.

²⁴⁷ <http://benwilbrink.nl/literature/hiele.htm>

Pierre van Hiele and Jan van de Craats

2015-09-06 ²⁴⁸

Jan van de Craats (University of Amsterdam) ²⁴⁹ wrote the textbook *All you need in maths!*, ²⁵⁰ using the UK “maths” instead of the USA “math”. The book need not fit a national curriculum and is presented as a book with exercises. The idea is to counter the trend in Freudenthal’s *realistic mathematics education* that forgets about decent practice and exercise.

I sent the following email to Van de Craats cc some other people involved in the Dutch discussion on mathematics education. The email speaks for itself. I take the liberty to include some weblinks for outsiders to the discussion. The original email contained fully stated URLs, but for readability on a web page I transform these in linked labels (in print with footnotes). The sections are made clearer. Some typo’s have been corrected. This weblog text closes with a comment that was not in the email.

The email

Date: Sun, 06 Sep 2015

To: “Craats, Jan van de” (UVA)

From: Thomas Cool / Thomas Colignatus

Subject: Inadequacy, maltreatment and abuse w.r.t. the work by Pierre van Hiele (1909-2010)

Cc: Persons mentioned below

Dear professor Van de Craats,

You are an informal leader of the movement amongst Dutch mathematicians to correct the so-called “didactics” of the Freudenthal Institute, which didactics [is] scientifically proven invalid but nevertheless dominates Dutch education in mathematics including arithmetic.

In the Dutch situation there is inadequacy, maltreatment and abuse w.r.t. the work by Pierre van Hiele (1909-2010). My intention is to inform you about this, because this helps for understanding the situation w.r.t. the Freudenthal Institute and mathematics education, and for identifying the direction for improvement.

(1)

Last year, 2014, the Dutch Academy of Sciences (KNAW) had a conference ²⁵¹ on education in arithmetic. I asked Jan Bergstra (UvA), ²⁵² secretary of the mathematics section ²⁵³ at KNAW to read Van Hiele’s “*Structure and Insight*” (in the Dutch original “*Begrip en Inzicht*”). I also asked him to support at Academic Press that they put out a new edition of this, and to fund an English translation of Van Hiele’s thesis. It took a while, but Bergstra now has reported that he read the book, and can do little with it. He seems to refer to his own interest in fractions (and division by zero), but that wasn’t the question. I expect a decent discussion at the KNAW math section about the crucial importance of Van Hiele’s work for math education, internationally. It is inadequate and a maltreatment that this section doesn’t have this discussion and evaluation, or did not report back to me so that I could see the quality of the argumentation. I cc to Jan.

²⁴⁸ <https://boycottholland.wordpress.com/2015/09/06/pierre-van-hiele-and-jan-van-de-craats>

²⁴⁹ <https://staff.science.uva.nl/j.vandecraats>

²⁵⁰ <https://staff.science.uva.nl/j.vandecraats/#allyouneed>

²⁵¹ <http://thomascool.eu/Papers/Math/2014-07-02-KNAW-Rekenen-deugt-niet.html>

²⁵² <https://staff.fnwi.uva.nl/j.a.bergstra/>

²⁵³ https://www.knaw.nl/nl/adviezen/adviesraden-en-adviescommissies/organisaties/287/akademiaorganisation_membersview

(2)

I asked Nellie Verhoef (TU Twente) what information she gave to David Tall (United Kingdom) about sources in Dutch about Van Hiele's work. I already spotted one crucial mistranslation w.r.t. the meaning of "realism" in "realistic mathematics education". Verhoef refuses to answer. David Tall appears to think that Van Hiele limited his theory of levels to geometry only. It would be David Tall who saw that they apply in general. This is a misconception, since Van Hiele indicated the general applicability already in his thesis of 1957. It is important however that Tall confirms the general value. Tall's book still requires a correction. It is crucial to know what information Nellie Verhoef gave him. It is a breach of the integrity of science that she refuses to disclose this information. I copy to Verhoef. I copy to Harrie Broekman (UU) who is connected to this issue. I reported the issue to Jan Bergstra in his capacity at KNAW, but he seems to neglect it. I copy to professor Mike Thomas²⁵⁴ [in New Zealand], so that he can check whether this email is relevant for David Tall (given his age and interest).

These two links give more information about the issue.

- Article on Van Hiele and Tall²⁵⁵
- Weblogtext on Freudenthal's fraud²⁵⁶

(3)

The thesis by S. la Bastide-van Gemert about Freudenthal contains some curious passages that Freudenthal took the theory of levels from Van Hiele and that Freudenthal himself was the inventor. I asked La Bastide what to make of this, and what her diagnosis about the origin was. She stated not to have time for this, in her current work at the Groningen Medical Center. Subsequently, I posed the same question to the thesis supervisors and readers, still at Academe so that it can be regarded as their work. I did this one by one, so not to overburden all. I informed each about the rejection by the predecessors. Each rejected to look into this. They neither fully and openly confirmed the inconsistency. But this is a breach in the integrity of science too. There is an inconsistency in a thesis, which one should not accept. There is all indication that Freudenthal stole the concept from Van Hiele, which is important to understand the full situation. It is unacceptable that this issue is covered up. I copy to La Bastide, thesis supervisors Klaas van Berkel²⁵⁷ en Jan van Maanen,²⁵⁸ en reader Martin Goedhart,²⁵⁹ all in Groningen. I reported the issue to Jan Bergstra in his capacity at KNAW, but he seems to neglect it.

The issue is documented in the appendix of my paper on [Van Hiele] and Tall, cited above.

The thesis by La Bastide is [here²⁶⁰].

(4)

There is the issue of retired psychologist Ben Wilbrink who discussed Van Hiele's theory of levels. I have asked Wilbrink to correct his misrepresentation, but he refuses to do so, and, what turns this into a breach of scientific integrity, refuses to explain why. Since Wilbrink is retired, I asked him whether he could mention a mediator who he would be willing to listen to. See my email to him below.

²⁵⁴ <https://www.math.auckland.ac.nz/~thomas/index/mt.html>

²⁵⁵ <http://arxiv.org/abs/1408.1930>

²⁵⁶ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

²⁵⁷ https://en.wikipedia.org/wiki/Klaas_van_Berkel

²⁵⁸ <http://www.uu.nl/staff/JAvanMaanen/0>

²⁵⁹ <http://www.rug.nl/staff/m.j.goedhart/>

²⁶⁰ <https://www.rug.nl/research/portal/publications/pub%2899b49f87-fc4b-4bd5-a568-d57d2c11b2f6%29.html>

I have documented the issue [*here*²⁶¹].

In sum, it is established beyond reasonable doubt that there is inadequacy, maltreatment and abuse in Holland w.r.t. the work by Pierre van Hiele (1909-2010).

Perhaps the problem is being caused by the “many hands” phenomenon, that there are many people involved and each individual is not aware of the impact of the sum total, but, still, if each maintained proper adherence to the rules of science, then there would have been no reason for this email.

One may hold that each case is an issue for the commissions of integrity at the separate universities, but my experience is that these don't function well, see how they treated the slander w.r.t. my book *Conquest of the Plane*, and see my letter to KNAW-LOWI on the collective breach on integrity:

- LOWI on COTP²⁶²
- LOWI and collective failure in integrity²⁶³

I copy to the board of the KNAW section on mathematics, excluding Johan van Benthem, who maltreated my work on logic when I was a student in econometrics in Groningen around 1980 and when I had a course in logic by Van Benthem. I kindly ask chairman Broer²⁶⁴ to forward this email to professores emeriti Van der Poel and [Zandbergen] for whom I cannot find an email address.

- Breach of integrity on logical paradoxes²⁶⁵

I copy to the president of KNAW, professor Van Dijk.²⁶⁶

I will put this email on my weblog.

Kind regards,

Thomas Cool / Thomas Colignatus
Econometrician and teacher of mathematics
Scheveningen, Holland
<http://thomascool.eu/>

Date: Sat, 05 Sep 2015
To: "Ben Wilbrink"
From: Thomas Cool / Thomas Colignatus
Subject: Kun je een bemiddelaar voorstellen ? (...)

Dag Ben,

At 2015-09-04, Ben Wilbrink wrote:

Ik wil dit niet, Thomas. Ik ga er niet op in.

Je zult gemerkt hebben dat ik een zeer tolerant persoon ben. Je negeert al jarenlang mijn kritiek op het onderwijs in wiskunde, en ik heb er weinig van gezegd. Ik respecteer ook je kennis en bijdragen.

Maar [...] t.a.v. je behandeling van Van Hiele maak ik nu groot bezwaar op grond van wetenschappelijke deugdelijkheid. Bij andere psychologen heb ik al opgemerkt dat ze te weinig van didactiek van wiskunde weten, en t.a.v. jou kan ik geen uitzondering maken.

²⁶¹ <https://boycottholland.wordpress.com/2015/09/05/pierre-van-hiele-and-ben-wilbrink/>

²⁶² <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

²⁶³ <http://thomascool.eu/Papers/Math/2014-07-08-Colignatus-aan-KNAW-LOWI.html>

²⁶⁴ <http://www.math.rug.nl/~broer/>

²⁶⁵ <http://thomascool.eu/Papers/AEOE/2015-05-21-A-breach-of-integrity-on-paradoxes.pdf>

²⁶⁶ <https://www.knaw.nl/en/about-us/organisatie/president-knaw>

Mijn tekst hierover:

<https://boycottholland.wordpress.com/2015/09/05/pierre-van-hiele-and-ben-wilbrink/>

Een oplossingstraject is dat je een bemiddelaar voorstelt, en ik kijk of ik akkoord ga.

Iemand voor wie je wel respect hebt en die jou hopelijk kan uitleggen in termen die je wel begrijpt dat deze zaken zijn op te lossen.

Met groet,

Thomas

Closing statement of this weblog entry w.r.t. the email

Van de Craats wrote the book with Rob Bosch (Netherlands Defense Academy).²⁶⁷ Bosch was member of the Social Choice Theory group that used false arguments to block²⁶⁸ my invited presentation in 2001 at the 37th Dutch mathematics conference (NMC), and discussion with Donald Saari. Bosch is also member of the team of editors of the journal *Euclides* for Dutch math teachers, that maltreated my books EWS and COTP, see here.²⁶⁹ I haven't looked at the contents of *All you need in maths!*, but it is reasonable to expect that it doesn't contain the didactic improvements suggested by EWS and COTP (and neither refers to those). Yes, when conventional math formats are crummy then you need more exercises to master them. While the true objective is to understand the math and not merely solve the sums.

(picture)

Jan van de Craats and his book *All You Need in Maths!* (source: his website)

²⁶⁷ https://nl.wikipedia.org/wiki/Nederlandse_Defensie_Academie

²⁶⁸ <http://thomascool.eu/Thomas/English/Science/Letters/SCT-working-group.html>

²⁶⁹ <http://thomascool.eu/Papers/COTP/LOWI/Index.html>

Jeroen Dijsselbloem

Jeroen Dijsselbloem on money and math

2013-03-27²⁷⁰

I wasn't going to join the hype in the European media about eurogroup president Jeroen Dijsselbloem. The hype is only about money and that isn't so interesting, contrary to what the nice people at the Financial Times tell you. However, it so happens that Jeroen Dijsselbloem also has been the chair of a Dutch *Parliamentary Inquiry Committee on Innovations in Education* in 2007-2008.²⁷¹ Now we are talking. For this brings us to the issue of the education in mathematics.

The European media hype is about whether Dijsselbloem knew what a '*template*' is. He says on Dutch television that he didn't know the word. He did however reply to a question containing it, see the FT Reuters transcript.²⁷² Apparently the political distinction is now being made between an 'approach' and a 'template'. We may figure that Dijsselbloem is a sensible and intelligent person who gets the drift of a question, so we can forgive him for not responding: "Can you explain what you mean by a *template*?" It is actually not so nice of the reporters at FT Reuters to make such a fuss about this. They are just as guilty in this blame-game, for after Dijsselbloem's reply they didn't ask for confirmation: "So this will be the template, just to be sure that we will not quote you in a wrong manner?" The real fuss is the command of English and the state of the Dutch system of education.

The European media hype is also about whether Dijsselbloem over-enthusiastically took the Dutch *approach* to SNS-Reaal-bank and Cyprus as the future *approach* for the Eurozone (if we allow for that word). It may be that he overplayed his position as president and that other members have different thoughts. This may indeed be the case. The other members may have shown polite interest in what Dijsselbloem has been explaining about his ideas, and it may be that Dijsselbloem mistook this for agreement. Deep in the hype, harsh words may have been spoken, but, as diplomats tend to do, internally, far removed from the spotlights. Well, every Dutch(wo)man has to learn that Europe isn't just a 'big Holland'. We all remember the difficulty that Wim Duisenberg had in 2000 after the introduction of the euro and the questions about the exchange rate policy.

Nevertheless, the EU is setting up a Banking Union with a European Banking Authority.²⁷³ On that single webpage we see words like 'supervision', 'regulation', 'mechanism', 'vision' and 'roadmap', but we do not see the word 'approach' yet. Downloading the Communication, we see that the Banking Union distinguishes 'the most significant European systemically important banks' and the 'others'. The first will have to be saved at all cost, the others will be allowed to implode. Thus Dijsselbloem quite accurately warns us to put our money in a system bank, unless we are risk-prone and like a little bit of higher interest plus the thrill of a possible collapse. The warning is also that each bank in trouble should quickly join a system bank so that it will be saved.

²⁷⁰ <https://boycottholland.wordpress.com/2013/03/27/jeroen-dijsselbloem-on-money-and-math/>

²⁷¹ http://nl.wikipedia.org/wiki/Parlementair_onderzoek_onderwijsvernieuwingen

²⁷² <http://blogs.ft.com/brusselsblog/2013/03/the-ftreuters-dijsselbloem-interview-transcript>

²⁷³ http://ec.europa.eu/internal_market/finances/banking-union/index_en.htm

I might sound a bit sarcastic but in reality I am referring to my earlier *Economic Plan for Europe*²⁷⁴ and the suggestion for a new EMU treaty,²⁷⁵ plus an additional analysis that I hope to be able to put on the web soon. [Addendum April 3: it is here now.²⁷⁶]

Now the interesting part. For Dutch education, we find a similar juggling of words. The Dijsselbloem Committee in 2008 distinguishes ‘what’ from ‘how’. Parliament decides what will be taught at school, and the teaching community decides how it will be taught. The Committee observes that this rule had been violated in the past, with various ‘innovations in education’, that Parliament loved but teachers abhorred, and that caused Dutch education to go down the drain.

The Committee didn’t investigate how it came about, that Parliament loved ‘innovations’ that the teachers abhorred. Apparently Dijsselbloem takes it for granted that Parliament doesn’t listen to teachers. Indeed, after the Committee report was declared a success, Parliament decided that highschool graduation should include a test on basic numerical skills, and a fail would even block graduation. Parliament thinks that this is a ‘what’ but actually it is a ‘how’. Learning to count is for young children and not for teenagers. Elementary schools should provide for those basic numerical skills, but they are failing to do so, both because of ‘innovations’ and because of elementary school teachers who have insufficient numerical skills themselves. Apparently Parliament wants to fix this by shifting the burden onto the higher level. Dutch readers with a strong heart and love for horror shows would want to read Jaap de Jonge “Opkomst en ondergang van de rekentoets” (*Rise and Fall of the Numerical Test*), March 2013, Euclides 88/5 p224-225.²⁷⁷ Unfortunately, that magazine of the Dutch Association of Teachers of Mathematics tends to keep important information behind a pay-wall.

Now to the Grand Finale. My point is that Parliament has decided that schools must teach math, but the teachers do not deliver math, but something that they call ‘math’. Mathematicians are trained for abstraction, but in class they meet real live students, and they resolve their cognitive dissonance by clinging to a tradition that has grown over the ages, but that isn’t targetted at proper didactics. See my books *Elegance with Substance* (EWS) 2009 and *Conquest of the Plane* (COTP) 2011. Thus the Dijsselbloem distinction at first seems to have some merit, but breaks down when Parliament refuses to check whether it really gets what it intends to get. The distinction between ‘what’ and ‘how’ is somewhat illusory, if the people responsible for the ‘how’ destroy the ‘what’.

The only solution is that Parliament starts paying attention to teachers rather than the bureaucrats and lobbyists. The only solution lies in an open atmosphere, where people can speak freely and frankly, where we treat people and ideas with respect, and where we judge issues on their merit.

More on this in my paper *What a mathematician might wish to know about my work*, March 2013.²⁷⁸

²⁷⁴ <http://mpira.ub.uni-muenchen.de/33476>

²⁷⁵ <http://mpira.ub.uni-muenchen.de/35120>

²⁷⁶ <http://mpira.ub.uni-muenchen.de/45759>

²⁷⁷ <http://www.nvww.nl/page.php?id=9040>

²⁷⁸ <http://thomascool.eu/Papers/Math/2013-03-26-WAMMWTKAMW.pdf>

The education of Jeroen Dijsselbloem

2013-04-09 ²⁷⁹

The Dutch minister of finance and eurogroup president Jeroen Dijsselbloem became (in-) famous in Europe overnight, with the handling of the crisis in Cyprus. He is decent, he is smart, and he belongs to the younger generation that is set to take over from Angela Merkel and François Hollande over the next decade. Indeed, it is better to call him Jeroen rather than Mr. Dijsselbloem and keep him as a household name, as common as that of a soccer player, not only since he will stay on the stage for a longer time but also because it is easier to communicate up close rather than over a long distance. If one thing has become clear from the Cyprus event is that Europe has to communicate a lot.

The Cyprus government had four years since 2008 to resolve its banking problem and had refused to do so. In September 2012 the EU presented plans for a European Banking Union. These plans distinguish between system banks that have to be saved at all costs and normal banks that may collapse. The Cyprus government could have read those plans, especially since Cyprus even was the EU-President in the second half of 2012. The Cyprus government still refused to resolve its problems and apparently hoped that the EU would provide ample funds. Alternatively said, the Cyprus banking elite didn't mind to use its own population as hostages, hoping that the bluff poker would work. The key message from the EU to Cyprus is that the EU did actually help, for otherwise the chaos would be much bigger. It is up to Cyprus now to get rid of its brutal elite, and find similar decent and smart people like Jeroen to clean up the mess.

It is good to know that Jeroen made an entrance on the political stage in Holland in about the same manner as now in Europe. He made his mark as the chairperson of a parliamentary enquiry into education. It is hard to believe, but the system of education in Holland had been spiralling downwards, and Jeroen suddenly was on national television, explaining what had gone wrong and what should be done to repair it. Overnight he was a national hero. A key thing to see is that this fame endured, and didn't disappear as sudden as it had come, as we see with so many hypes nowadays. Jeroen is there to stay.

Europe has had strange political leaders, like Helmut Kohl from Germany who got lost in an illegal party financing deal and Jacques Chirac from France who got convicted to two years of prison for diverting public funds. With leaders like that, you may question what they say or do. Think of Cyprus again, or of Berlusconi. The Greek who hate Angela Merkel should rather protest against Berlusconi and his lame promises. In any case, with Jeroen, we do not need to fear for this. Jeroen isn't perfect and may make errors, but given that he is decent and smart, an error still means that no mischief is intended, so that he can learn from mistakes, and he tends to be open to new information that can help to correct such errors.

The key message of this article is that there is new information. Up to now, economic science has been explaining that a common currency like the euro also requires a political union. This is called the theory of the optimal currency area. In the Treaty of Maastricht of 1992, the political leaders created the euro but rejected the political union. The euro indeed was badly managed. Europe now goes from crisis to crisis, potentially making the minds of the population ripe to accept a United States of Europe, and give up national sovereignty, but also with the risk of rising nationalism and the break-up of what already exists.

Now, however, there is a new economic theory, that would allow to maintain both the euro and national sovereignty, provided that each nation adopts its own national Economic Supreme Court (ESC), that supervises national economic policy. The national

²⁷⁹ <https://boycottholland.wordpress.com/2013/04/09/the-education-of-jeroen-dijsselbloem/>

ESC would be staffed by national economic scientists, who know their nation better than distant Brussels, and who operate under the rules of science, rather than the hidden rules of a distant bureaucracy. The international co-ordination that is required for a common currency comes about by the international scientific co-operation of the various ESCs, that is transparent and open to science and the public. This new economic theory plus a scheme how to handle the euro is discussed in the paper "*Money as gold versus money as water*" available at the Munich archive for economic papers.

Since I am Dutch and present this analysis from Holland, and since Jeroen is Dutch and is the minister of finance of Holland, readers may think that Jeroen and I have discussed this new theory, and that he is well-informed on this. Unfortunately, it doesn't work like that. A huge unemployment and political chaos across Europe and complex communications and diplomacies over a few thousand kilometers apparently are required, to cross the few kilometers from my desk to his desk. There is also censorship of science in Holland since 1990. The Dutch Central Planning Bureau (CPB) is not an Economic Supreme Court and does not apply the proper rules of science. Next to 'Greek statistics' there is 'Dutch economic science'. Thus Jeroen entertains a misperception about the quality of his economic advisors and he lacks information about the new economic theory. Thus Europe needs some education on Jeroen, and Jeroen needs some education on Europe. The best thing that Europe can do is help Jeroen gain more information indeed where this is lacking now.

Thomas Colignatus is an econometrician and teacher of mathematics in Scheveningen, Holland. The article "Money as gold versus money as water" is available at <http://mpa.ub.uni-muenchen.de/45759/>

Dijsselbloem on Dutch exports

2013-08-23 ²⁸⁰

Eurogroup president Jeroen Dijsselbloem is not a macro-economist. By training he is an agricultural economist. ²⁸¹ He will rely on macro-economists and judge their advice by economic common sense. That common sense may however also be influenced by traditional mercantilist ideas that exports will earn us gold and make us rich. There is also the success of Dutch agricultural exports that may cloud his judgement.

His recent letter of June 19 to Dutch Parliament about the budget deliberations for 2014 contains this key statement (p3): ²⁸²

“Holland is in a so-called debt-recession. (...) The usual pattern of economic recovery in Holland (rising exports, rising investments, rising consumption) is slowed down by the type of crisis.” (In Dutch gibberish: “Nederland bevindt zich in een zogenoemde balansrecessie. (...) Het gebruikelijke patroon van economisch herstel in Nederland (export trekt aan, investeringen groeien en particuliere consumptie neemt toe) komt door de aard van de crisis langzamer tot stand.”)

But the export surplus in 2013 is about 10% of GDP, see the CPB Central Economic Plan. ²⁸³

The Dutch surplus contributes to the deficits of Southern Europe. The huge Dutch surplus is part of the European problem. Christine Lagarde of IMF shouldn't send a team only to Greece but also to Holland, to sternly explain that the situation is intolerable and that Holland should work towards a balance (notably by importing more).

Yes, there is a debt crisis. If you insist on recovery via exports then recovery will be slow. But the debt crisis does not prevent you from tackling the export surplus. Full employment can be restored at home, by proper internal measures.

Dijsselbloem gets his advice from macro-economists who have been emphasizing exports since 1970. The Dutch system of social security creates a huge unemployment on the home market. The Dutch solve this by lower wages and relying on exports. The Dutch have been exporting their unemployment since 1970.

See my 1996 paper on the exposed and sheltered sectors, at EconPapers ²⁸⁴ or a local file with graphs. ²⁸⁵ It is also a chapter in DRGTPE ²⁸⁶ and an update is in CSBH. ²⁸⁷

The Eurozone group has a president who is one of the key creators of the problems that they discuss, though sadly enough they aren't aware of this. (That is, they may think that he causes other problems.)

(picture: Eurogroup March 15 2013, Schauble, Lagarde, Dijsselbloem (Source: EU Council))

²⁸⁰ <https://boycottholland.wordpress.com/2013/08/23/dijsselbloem-on-dutch-exports/>

²⁸¹ http://en.wikipedia.org/wiki/Jeroen_Dijsselbloem

²⁸² <http://www.rijksoverheid.nl/ministeries/fin/documenten-en-publicaties/kamerstukken/2013/06/19/hoofdpijnenbrief-begroting.html>

²⁸³ <http://www.cpb.nl/publicatie/centraal-economisch-plan-2013>

²⁸⁴ <http://econpapers.repec.org/paper/wpawuwpgt/9608001.htm>

²⁸⁵ <http://thomascool.eu/Papers/ShelteredExposed/AGEshex.html>

²⁸⁶ <http://thomascool.eu/Papers/Drgtpe/Index.html>

²⁸⁷ <http://thomascool.eu/Papers/CSBH/Index.html>

Part 3.

International: Feeling the water

General public, philosophy and history

Amir Alexander and history as storytelling

2014-06-14 ²⁸⁸

Amir Alexander ²⁸⁹ is a historian at UCLA and has written some books that have drawn attention. I haven't read them but only summaries and some reviews. Alexander's *modus operandi* apparently is: find a good narrative and weave history around it. The story sells, and history hitch-hikes along.

For example Alexander's book *Geometrical Landscapes: The Voyages of Discovery and the Transformation of Mathematical Practice* ²⁹⁰ uses the narrative of exploration and imposes this on mathematics too. Reuben Hersh debunks it. Be sure to first read Davis & Hersh, *The Mathematical Experience*, ²⁹¹ and then check out Hersh's review. ²⁹² Hersh quotes Alexander:

"I find a narrative approach to be a most promising avenue for historicizing mathematics. (...) Mathematical work does, I argue, contain a narrative. Once this narrative is identified, it can be related to other, nonmathematical cultural tales that are prevalent within the mathematicians' social circles."

Hersh finds this okay but indicates that Alexander goes too far in suggesting that such narratives are also driving forces within mathematics. Reuben Hersh: ²⁹³

"If there existed a prevalent social or cultural story that was analogous or parallel to the mathematical story, it by no means follows that such a story "shaped" or "guided" the mathematics. Such a social or cultural story may have simply served as a model for how one talked about or advertised the mathematics." (American Scientist)

Exactly.

Alexander recycles narratives and weaves history around it, and for some reason he has chosen mathematics to be his victim.

Duel at dawn

We see this even clearer in his earlier book *Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics*. ²⁹⁴ Check these reviews:

(1) Reviel Netz: "Everyone is unhappy after his or her own fashion, but the font of possible stories is limited. Hence stories tend to flatten the many particular sorrows into a few single types of narrative misery. Amir Alexander is interested in one such type: the misery of the Outsider Mathematical Genius." (Common Knowledge) ²⁹⁵

²⁸⁸ <https://boycottholland.wordpress.com/2014/06/14/amir-alexander-and-history-as-storytelling/>

²⁸⁹ <http://www.history.ucla.edu/people/faculty/faculty-1/faculty-1?id=864>

²⁹⁰ <http://www.sup.org/book.cgi?id=1387>

²⁹¹ <http://www.ams.org/notices/199710/comm-millett.pdf>

²⁹² <http://www.americanscientist.org/bookshelf/pub/in-search-of-interior-riches>

²⁹³ <http://www.americanscientist.org/bookshelf/pub/in-search-of-interior-riches>

²⁹⁴ <http://www.hup.harvard.edu/catalog.php?isbn=9780674046610>

²⁹⁵ http://muse.jhu.edu/login?auth=0&type=summary&url=/journals/common_knowledge/v017/17.3.netz.html

(2) Daniel S. Silver: “Mathematicians might have been the only scientists who acquired the popular label of heroic misfit, but *Duel at Dawn* can leave readers with the impression that most mathematicians wore the unhappy brand. Besides Galois, we find Abel and Bolyai offered as examples, and, arguably, Cauchy, too. Yet there were many well-known mathematicians in the nineteenth century who were seen as anything but misfits: Cayley, Dirichlet, Gauss, and Hamilton are just a few names that come to mind. And there are plenty today. So why are sad tales of unstable mathematicians from Galois to Perelman so popular? *Duel at Dawn* suggests a reason. The public sees mathematicians as being like artists, preferring to live in a virtual world that bears little resemblance to what G. H. Hardy called “this stupidly constructed ‘real’ one”. Such people cannot possibly be happy or sane. Another reason, one not offered by Alexander, can be found in our public high schools.” (AMS Notices)²⁹⁶

Translate this as: A so-called “historical” tale relies on current (bad) education to sell well.

(3) Michael Patrick Brady: “(...), author Amir Alexander argues that the popular image of mathematicians as strange, reclusive figures springs from the early 19th century. It was then that mathematics began to evolve from a science based in the empirical realities of the Enlightenment to an art form informed by the ideals of Romanticism, concerned only with its own internal truths.” (Forbes)²⁹⁷

Translate this: Brady fell into Alexander’s trap. Brady never heard about the distinction between Greek math and engineering ? Brady neither has the criticism on the math as given by Silver.

Infinitesimal

Now in 2014 Alexander published this new book *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World*.²⁹⁸

(a) National Public Radio has a nice article²⁹⁹ and interview.³⁰⁰ Indeed, start by being nice, they aren’t that critical as I will be below.

From Dirk Struik’s history of mathematics, I already knew that around 1600 most universities were quite conservative and that over the next century new academies were created for the new demands from society and trade and industry. Now Alexander reports that five leaders of the Society of Jesus convened on August 10 1632 and actually forbade the use of indivisibles (later giving rise to infinitesimals). Alexander claims that the Jesuits succeeded in keeping Italy in the Euclidean mold. What I regarded as common human conservatism thus would actually be *organised*. The countries that were relatively free from Rome also allowed for more freedom in mathematics, whence they discovered new methods, like the infinitesimal. Henri VIII (1491-1547) and his six marriages thus indirectly contributed to the creation of calculus.

(b) Judith V. Grabiner has a review at MAA.³⁰¹ An important point:

“Aristotle, arguing for the potentially infinite divisibility of the continuum, had explicitly ruled out both indivisibles and the actual infinite.”

²⁹⁶ <http://www.ams.org/notices/201010/rtx101001297p.pdf>

²⁹⁷ <http://www.forbes.com/2010/04/13/mathematics-ammir-alexander-millennium-prize-opinions-book-review-michael-patrick-brady.html>

²⁹⁸ <http://us.macmillan.com/infinitesimalhowadangerousmathematicaltheoryshapedthmodernworld/>
AmirAlexander

²⁹⁹ <http://www.opb.org/news/article/npr-far-from-infinitesimal-a-mathematical-paradoxs-role-in-history/>

³⁰⁰ <http://www.npr.org/2014/04/20/303716795/far-from-infinitesimal-a-mathematical-paradoxs-role-in-history>

³⁰¹ <http://www.ww.amc8.org/publications/maa-reviews/infinitesimal-how-a-dangerous-mathematical-theory-shaped-the-modern-world>

Yes, in the continuum there are no indivisibles. Yes, for a line going to infinity there is no reason to hold that it is an actual infinite (since what would that be?). However, it would be strange to say that the continuum, say the interval $[0, 1]$, cannot be divided infinitely (or indefinitely). In terms of division, the continuum is an actual infinite. If "actual infinite" is to have any meaning, it is the continuum. It would be strange to think that Aristotle thought otherwise. I haven't studied Aristotle here though.

[2015: Aristotle,³⁰² Physics III, Part 6 gives: "Further, a thing is infinite either by addition or by division. Now, as we have seen, magnitude is not actually infinite. But by division it is infinite. (There is no difficulty in refuting the theory of indivisible lines.) The alternative then remains that the infinite has a potential existence." and "Our definition then is as follows: A quantity is infinite if it is such that we can always take a part outside what has been already taken. On the other hand, what has nothing outside it is complete and whole." The latter definition chooses the potential infinite, and this fits Grabiner's observation. However we can also observe the idea that the continuum is an actual infinite, as $[0, 1]$ is a whole.]

I still allow for the possibility that the Jesuits were right. Calculus may have been discovered by Newton's use of infinitesimals, but Cauchy developed it with limits, in order to solve the mathematical problems that he and others saw w.r.t. infinitesimals. Grabiner's critique: "Alexander's book contains occasional imprecise statements. Notably, although the book's main title is "Infinitesimal," the Jesuit condemnations he quotes denounce indivisibles, not infinitesimals."

Thus I fully agree with Grabiner:

"But I think Alexander overestimates the importance of the disputes he describes. He doesn't show that the Jesuit condemnation of indivisibles was anywhere near as influential as the Church's condemnation of Galileo's Copernicanism. Nor does he show that the ideas of Cavalieri and Wallis were widely seen as dangerous and disruptive. And, since a key point of Alexander's book is the importance of the invention of the calculus, I'd argue that the calculus would still have been invented in the seventeenth century even had the Jesuits convinced everyone in Europe that indivisibles were not rigorous mathematics." (MAA)

But also Grabiner falls in Alexander's trap: "Nonetheless, the stories Alexander tells about these disputes are fascinating, and they deserve to be better known." I would rather hold that the truth be known, and not the stories that Alexander tells.

Alexander thus surprises me on the organised conservatism in Italy. But it seems that he puts too much emphasis on the power of organisation in other countries. Given such freedom, there still was room for good old common human conservatism that doesn't need organisation. While Newton discovered calculus and derived his laws of gravity using calculus, he still presented his Principia with Euclidean methods. To see Alexander's evidence I would need to read his book, but his *modus operandi* makes me wary. My fear is that I will just read a common narrative of freedom against tyranny, adorned with elements from the history of calculus to justify his job contract at UCLA.

(c) Check Alexander's own text in the Scientific American followed by critical comments by readers.³⁰³ His final paragraph reads:

"By transforming the calculus into a rigorous mathematical system, Cauchy ended a conflict that had lasted more than two millennia. In the 5th century B.C.

³⁰² <http://classics.mit.edu/Aristotle/physics.3.iii.html>

³⁰³ <http://www.scientificamerican.com/article/a-brief-history-of-infinitesimals-the-idea-that-gave-birth-to-modern-calculus/>

Hippasus had shown that mathematics could never fully describe the world. In the 19th century A.D. Cauchy showed that it didn't have to: Mathematics would survive, and thrive, on its own, freed from the shackles of material reality. Modern mathematics was born."

This is a crooked paragraph. Mathematics has always been free from the shackles of material reality. Mathematics namely is abstraction. Hippasus didn't do what Alexander claims he did. A statement like "Modern ... was born" is story-telling and an empty shell.

(d) The review by Alan Hirschfeld,³⁰⁴ professor of physics, is informative, but: "That a mathematical theory can be characterized as dangerous, much less world-shaping, pings my skeptic's radar. To earn this dual distinction for what appears to be merely a centuries-old quibble over the nature of points, lines and planes sets a formidable test for any author. But Mr. Alexander succeeds, weaving the strands of a colossal mathematical dispute into the fabric of Western cultural history. The result is an interpretive tapestry whose richness justifies his exclamatory subtitle." (Wall St. Journal)

Translate this as: Another victim fell into the trap.

Note that infinitesimals were still a problem at the time of Cauchy. It is somewhat strange to have infinitesimals in the title when the story ends with limits. The true umbrella is the birth of calculus, but Alexander cannot tell that story, since he wants to focus on infinitesimals since the Jesuits forbade indivisibles. The general readership will be interested in the struggle of science against tyranny. To relive that same old narrative, they now are treated on somewhat crooked mathematics and somewhat crooked history of mathematics.

People should be able to ask their money back if a book does not deliver what is promised.

Of course, my book *Conquest of the Plane* shows that calculus can be developed with algebra, without limits and without infinitesimals. But I don't give a money-back-guarantee, because traditionally minded mathematicians have shown that they don't read well, see the earlier entry.³⁰⁵

NB 1. In a way I feel a bit vindicated that the narrative 'democracy and mathematics' gets such attention. I have been advocating that his combination gets more attention. However, the better narrative is:

- mathematics is liberating itself, since no authority can force you to accept a theorem except your own understanding of the proof
- democracy deteriorates when people get bad education in mathematics
- mathematicians have been destroying democracy since Kenneth Arrow presented his 'impossibility theorem' in 1950 / 1951, see my book *Voting theory for democracy*.

NB 2. Everything hangs together. For math education, see these suggestions for improvement:

- the book *Elegance with Substance* (2009)
- the book *Conquest of the Plane* (2011)
- this note on infinity³⁰⁶ - now superseded by FMNAI (2015)
- all leading to *Neoclassical mathematics for the schools*³⁰⁷ - idem.

³⁰⁴ <http://online.wsj.com/news/articles/SB10001424052702303380004579521490104668518>

³⁰⁵ <https://boycottholland.wordpress.com/2014/06/12/slander-squared/>

³⁰⁶ <http://thomascool.eu/Papers/ALOE/2012-03-26-CCPO-PCWA.pdf> - now superseded by FMNAI (2015)

³⁰⁷ <http://thomascool.eu/Papers/Math/2011-09-06-NeoclassicalMathematics.pdf>

Math philosophy for a general audience

2014-06-03 ³⁰⁸

Rob Nanninga (1955), editor of the Dutch skeptic magazine *Skepter*, passed away suddenly on May 30. ³⁰⁹ I had some contact with him about my book *The simple mathematics of Jesus* (SMOJ) (2012). *Skepter* publishes skeptical articles against astrology and is hesitant to discuss religion, so that my analysis that the Bible is an astrological book puts the editors in a difficult spot. *Skepter* hasn't published about SMOJ yet, but I tend to think that Nanninga was pondering it seriously. He died while working on an article on Scientology – by believers called a religion.

Looking for his obituary brought me to one of the skeptic websites, where I noted a webcast by Keith Devlin ³¹⁰ about the 10 myths about the Golden Ratio, ³¹¹ i.e. the number $\phi = 1.61\dots$ (*phi*).

Again, we see the Dutch language working out as a dungeon sink, since all kinds of English materials are put on that Dutch website but foreigners will not be able to follow the discussion. The Dutch skeptics are very open to foreign material but hesitant about home-grown SMOJ. ³¹²

Nevertheless, it is a Small World, and everything hangs together. Let us make a table:

<i>Devlin & Livio</i>	<i>Me</i>
Professor Devlin also refers to the book about the Golden Ratio by Mario Livio. ³¹³	I discussed an aspect of phi, see Pyramids and the Meter. ³¹⁴ Apparently that aspect isn't in Devlin's list, and I wouldn't know whether it is in Livio's book.
Professor Livio also wrote a book <i>Is God a Mathematician?</i> ³¹⁵ – or see this other review. ³¹⁶ As I understand it, Livio refers to God metaphorically like Albert Einstein did when saying "God doesn't use dice" (no exact quote). His book really is about the 'unreasonable effectiveness of mathematics' (Wigner).	(a) This is a different subject than SMOJ that proposes a multidisciplinary project, see its page. (b) In my view, mathematics isn't <i>unreasonably</i> effective. It is reasonably effective since we choose it to be so. But please note that highschool mathematics education can be quite unreasonable, see EWS. (A reference on the effectiveness of mathematics is the Davis & Hersh book, <i>The mathematical experience</i> , ³¹⁷ see the AMS Review page

³⁰⁸ <https://boycottholland.wordpress.com/2014/06/03/math-philosophy-for-a-general-audience/>

³⁰⁹ <http://www.gezinsbode.nl/hoofdredacteur-en-scepticus-rob-nanninga-van-de-skepter-overleden/>

³¹⁰ <http://www.stanford.edu/~kdevlin/>

³¹¹ <http://kloptdatwel.nl/2014/03/15/tien-mythes-de-gulden-snede/>

³¹² <https://boycottholland.wordpress.com/2015/02/02/the-closed-dutch-mind-on-jesus-too/>

³¹³ <http://www.mariolivio.com/about-the-author/>

³¹⁴ <https://boycottholland.wordpress.com/2013/01/27/pyramids-and-the-meter/>

³¹⁵ <http://www.washingtonpost.com/wp-dyn/content/article/2009/02/05/AR2009020502876.html>

³¹⁶ <http://shinybookreview.com/2011/07/24/mario-livios-is-god-a-mathematician-math-as-philosophy/>

³¹⁷ http://en.wikipedia.org/wiki/The_Mathematical_Experience

³¹⁸ <http://www.ams.org/news/math-in-the-media/reviews>

	too, ³¹⁸ see my view in FMNAI 2015.)
A lecture by Keith Devlin: <i>How Did Human Beings Acquire the Ability to do Math?</i> ³¹⁹ – which Mario Livio might translate as: <i>How did Human Beings become like Gods ?</i> Devlin actually discusses his book <i>The Math Gene</i> – see this AMS Review by Allyn Jackson. ³²⁰	See my paper, included in both SMOJ and FMNAI: <i>Education of mathematics and brain research.</i> ³²¹ This seems a bit more concrete on how students can learn mathematics. My suggestion is that philosophy can run astray and that it is better to use empirical didactics as the whetstone.
Lectures by Keith Devlin are: <i>The Birth of Algebra</i> ³²² & <i>Calculus: One of the most successful technologies.</i> ³²³	See my video: <i>The algebraic approach to the derivative</i> ³²⁴ or see the sheets. ³²⁵ This thus combines these angles.
Lectures by Keith Devlin on (1) voting theory in the 2 nd half of above “birth of algebra” and (2) the birth of probability theory by Pascal & Fermat in the 2 nd half of above “calculus”.	See (1) my book <i>Voting theory for democracy</i> , and (2) at another time I must return to that issue of Pascal & Fermat.

These Devlin Lectures don't seem to be intended for who already knows math. They seem to be for a general audience like first year students who want a philosophical overview. The lectures have a little bit of actual mathematics but this is presented so fast and so superficial that new students will not get it. The lectures might be typical for mathematicians who hardly have insight in the education in mathematics. However, knowing or assuming that his intended audience will not be able to get real math, professor Devlin likely chooses to tell stories that somewhat convey some general impressions. By consequence, we now have 'math philosophy for a general audience' which somewhat translates as 'celibacy philosophy for adolescents'.

Good video presentations also have a PDF printout, but these seem to be lacking here. I am amazed that I actually managed to watch as much as I did, but the point seems to be that you can only judge on videos when you have actually seen them. Sobered by Rob Nanninga's passing away I patiently sat through professor Devlin's lectures on phi, the *Math Gene* and the parts on Algebra and Calculus (skipping voting theory). I am sorry to say that I tend to agree with some of the critical remarks on the *Math Gene* lecture: “Usually mathematicians are direct and they cut to the chase. (...) He got me really bored” and “This guy is the king of jibber jabber. Talks for minutes and says 2 things.” Other reactions have been positive but those are less revealing about their background and seem from general viewers. It does seem indeed that the lectures might be appealing to such a general audience, but for an institute like Stanford I would require much more quality.

I did enjoy professor Devlin's explanation about algebra however, since it nicely fits my suggestion that the derivative belongs to algebra. In the lecture on calculus he explains it standardly in terms of limits, but this focusses on numbers, instead of reasoning logically and qualitatively about numbers.

³¹⁹ <http://www.youtube.com/watch?v=NnVubBrATIU>

³²⁰ <http://www.ams.org/notices/200102/rev-devlin.pdf>

³²¹ <http://thomascool.eu/Papers/Math/2011-07-11-COTP-Damasio.pdf>

³²² <http://www.youtube.com/watch?v=FME9avU3u2Y>

³²³ <http://www.youtube.com/watch?v=8ZLC0egL6pc>

³²⁴ http://www.youtube.com/watch?v=gn_BKZaDa-o&feature=youtu.be

³²⁵ <http://thomascool.eu/Papers/COTP/2013-11-11-ColignatusStudiedagNVVW-English.pdf>

What is algebra?

- ✦ Algebra involves thinking **logically**, rather than **numerically** (as in arithmetic).
- ✦ In arithmetic you reason (calculate) **with** numbers; in algebra you reason (logically) **about** numbers.
- ✦ Arithmetic involves **quantitative** reasoning **with** numbers; algebra involves **qualitative** reasoning **about** numbers.
- ✦ A key feature of algebra is you introduce a name for an **unknown** and reason logically to find its value.
- ✦ Substituting numbers in an algebraic formula to obtain a numerical answer is **arithmetic**, not algebra; **deriving** that formula in the first place is algebra.

5:35 / 1:44:22



Keith Devlin, *The Birth of Algebra*, min. 5:35

Furlough on Frenkel

Edward Frenkel didn't study Math Education

2014-10-12 ³²⁶

I presume that professor Edward Frenkel ³²⁷ has contributions to research mathematics (RM), though I am not in RM and not qualified to judge. Being a teacher of mathematics myself and a proponent of re-engineering math education, see my book *Elegance with Substance* (EWS), ³²⁸ I am sympathetic to his background as a refugee from Russia (with perhaps often better mathematics than the USA) and his endeavour to enhance the quality of mathematics education in both Russia and the USA.

My problem is that he is a research mathematician (RM) and not a mathematics education researcher (MER). Thus the same problems return that EWS already exposed. Edward Frenkel is a best-selling media-star with the charisma of Sean Connery preaching however a somewhat perverted story on education. He is probably full of good intentions but deludes himself and the audience.

Paraphrasing Sean Connery's film "*From Russia with love*" ³²⁹ Frenkel explains that he is from Russia with both math and love. In the Stephen Colbert Show, ³³⁰ after discussing the fractions of vodka drinking, Colbert asks (minute 4): "Is it 2+2 or whatever Putin says it is?" Frenkel dodges the question by saying that he cannot speak "for Putin". But that wasn't exactly the question, and it is remarkable that a mathematician isn't precise.

(Picture: Colbert asking Frenkel: "Is it 2+2 or whatever Putin says it is?" Source: show video.)

In the *Los Angeles Times* opinion piece "*How our 1,000-year-old math curriculum cheats America's kids*", ³³¹ it is striking that he isn't precise again. Of course the curriculum has changed in the last 1000 years. For example, derivatives were included. More to the point, however, the USA had its 'math wars', ³³² partly provoked by the Russian Sputnik. In one video online, it is indeed asked whether media-star Frenkel himself would be the new Sputnik that alarms the USA to get its act together.

Overall, it is a wrong diagnosis to hold that the curriculum wasn't changed. The true diagnosis is that it are research mathematicians (RM) like Edward Frenkel himself who have no idea about empirical education who continue to throw monkey wrenches into the discussion on mathematics education (ME). EWS advises the parliaments of the world to have investigations to break the stranglehold of these nutcase research mathematicians. What is necessary is to re-structure the mathematics industry so that education is recognised as an empirical science. See the approach of using the model of the medical school. ³³³

³²⁶ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

³²⁷ <http://math.berkeley.edu/~frenkel/>

³²⁸ <http://thomascool.eu/Papers/Math/Index.html>

³²⁹ <http://www.imdb.com/title/tt0057076/>

³³⁰ <http://thecolbertreport.cc.com/guests/edward-frenkel/xj9d66/edward-frenkel>

³³¹ <http://www.latimes.com/opinion/commentary/la-oe-adv-frenkel-why-study-math-20140302,0,5177338.story> \l "axzz2ujmGs1i9"

³³² http://en.wikipedia.org/wiki/Math_wars

³³³ <https://boycottholland.wordpress.com/2014/07/18/the-medical-school-as-a-model-for-education/>

Who watches the video's of professor Frenkel's performances will be appalled, or should watch till one is. He uses "mathematics" in every other sentence as if this would explain anything. Overall, the audience sits up and looks like rabbits in the searing lights of the nutcase but sexy young professor. "Why are you speaking about love, I *hate* mathematics," Colbert asks (paraphrased). Frenkel provides a silly rationale: "You have been taught painting fences, not painting masterpieces" (paraphrased).

So, please explain, professor Frenkel: (a) Everyone can be taught how to paint a masterpiece ? (b) And there is no need for you to look into the empirical science of education of mathematics, and first write some reports on that, before you commit the fraud and crime of a breach in the integrity of science that you should not claim results that you are not qualified for ?

Edward Frenkel's abuse of love isn't love

2014-10-12³³⁴

There is no denying that Edward Frenkel³³⁵ loves mathematics and loves telling other people about it. We can be sure that he loves some people, family and colleagues, since he comes across as perhaps even charming, and it is difficult to become so charming if you wouldn't love some people – psychopaths excluded.

See for example these three video's currently also on his website: (a) the Colbert show,³³⁶ (b) with Chris Carter at the Los Angeles Library Foundation,³³⁷ (c) with Eric Weinstein at the Speyer Legacy School.³³⁸ I pity you though if you feel that you have to sit through all sessions as I did, see my first criticism.³³⁹

However, we cannot infer that Edward Frenkel loves other people indiscriminately, alas. That would be a different issue. His book title *Love and Math: The Heart of Hidden Reality*³⁴⁰ may well be deceptive since it has religious and literary tunes that do not fit science and its popular science expositions.

In loving everyone, it even is harder to define what "love" would mean. It is perhaps awkward to refer to Vladimir Putin again, who will be surely loved by many Russians, but who will not immediately spring to mind to many in the EU or USA when considering Frenkel's proposition, except when we dwell a bit longer on the *hidden reality* issue.

A definitely more serious author on love is Dewanand, "*A thousand ways to love*".³⁴¹ Earlier I favourably reviewed Dewanand's "*Holland: Paradise or Hell?*".³⁴² His other book on love has a review that I agree with.³⁴³ Thus, if you are interested in love, read Dewanand, and, if you must, consider Edward too.

(picture)

Edward Frenkel is a seriously confused person. His argument is that people should discover the beauty of mathematics. He accepts that people will not do so, since apparently the system of education has gone wrong. Instead, Frenkel decides to use the emotional rather than the cerebral approach (*Slate*³⁴⁴). Thus, he doesn't present the beauty of mathematics but some concoction that merely suggests it. It beats my mind how this could ever convince anyone, except that it shows to everyone again that research mathematicians are serious nutcases.

"So much damage has been done in terms of the way mathematics is misunderstood by our society. It has essentially become impossible to talk to most people directly about it. With the film *Rites of Love and Math*,³⁴⁵ which I

³³⁴ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkels-abuse-of-love-isnt-love/>

³³⁵ <http://math.berkeley.edu/~frenkel/>

³³⁶ <http://thecolbertreport.cc.com/guests/edward-frenkel/xj9d66/edward-frenkel>

³³⁷ <http://www.lfla.org/event-detail/936/Edward-Frenkel-and-Chris-Carter>

³³⁸ <http://www.speyerlegacyschool.org/page.cfm?p=442&newsid=35>

³³⁹ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

³⁴⁰ http://www.amazon.com/gp/product/0465050743/ref=as_li_ss_tl?ie=UTF8&camp=1789&creative=390957&creativeASIN=0465050743&linkCode=as2&tag=slatmaga-20

³⁴¹ <http://sbpra.com/Dewanand/>

³⁴² <https://boycottholland.wordpress.com/2014/05/16/review-and-praise-for-holland-paradise-or-hell-q-by-dewanand-2010/>

³⁴³ <http://www.pacificbookreview.com/thousand-ways-love-update-kama-sutra/>

³⁴⁴ http://www.slate.com/articles/health_and_science/new_scientist/2013/10/edward_frenkel_on_love_and_math_what_is_it_like_to_be_a_mathematician.single.html

³⁴⁵ <http://www.newscientist.com/blogs/culturelab/2010/04/erotic-equations-love-meets-mathematics-on-film.html>

made four years ago with the French director Reine Graves, the idea was to penetrate some of those defenses—to talk about math indirectly by appealing to the emotional rather than to the cerebral.” (Frenkel in *Slate* ³⁴⁶).

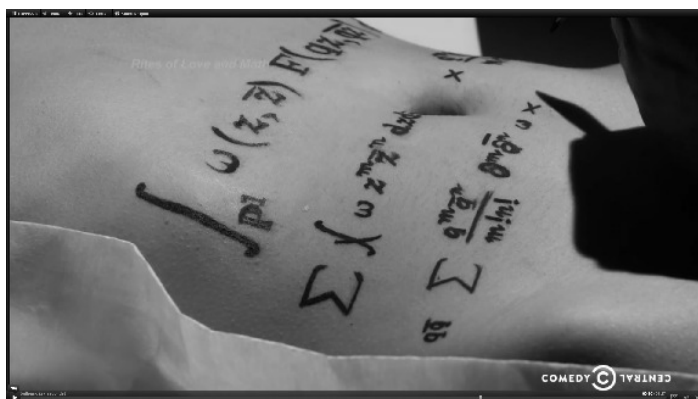
Another step in Frenkel’s delusion is the way how he links mathematics to art, and in particular mathematician Kurt Gödel to Vincent van Gogh – see the Speyer video, after minute 60. See my book ALOE or this brief exposition ³⁴⁷ that Gödel was deluded himself. I am much in favour of good mathematics education so that the world can get rid of the delusions by Kurt Gödel and the cult of logicians around this. Van Gogh remains a fascinating artist, with the advantage that everyone can see this for himself or herself.

Another step in Frenkel’s delusion is that he adopts the idea of tattooing a message onto a woman’s skin. This idea was already used by Ayaan Hirsi Ali, in the film “*Submission*”, ³⁴⁸ directed by Theo van Gogh, nephew of the painter, assassinated here in Holland on November 2 2004, soon 10 years ago.

When seeing that formula in the trailer, my first question was: “*What does that formula mean ?*” But the trailer doesn’t provide any clue. In the *Slate* interview, Frenkel says:

“In the film, we show a formula being tattooed onto the skin, becoming part of the body, and we hoped that this would allow the viewer to think about mathematics differently. The best reward was when somebody from the audience would raise his or her hand and ask, “But what does the formula mean?” which was exactly the point.”

I don’t see the point. Why not put the formula on a blackboard and not torture the poor damsel ? Why not start out with what it would mean, so that we can check that it is correct without disturbing the poor damsel ? Why think that the emotional cabal will cause improved education in mathematics ? Why would it cause people to study that supposedly improved education in mathematics ?



Formula on body, from trailer of “Rites of Love and Math” (Source: trailer)

In the *Los Angeles Library Foundation* video, we hear that Edward Frenkel is a Platonist, who thus believes that mathematical ideas have some form of “objective” existence out there just waiting to be discovered (unless he is confused about this). Surprisingly, Chris Carter, the creator of *The X-Files* ³⁴⁹ turns out to believe in God (not further specified) (unless he is confused about this). It must be mentioned that *The X-Files* are a form of

³⁴⁶ http://www.slate.com/articles/health_and_science/new_scientist/2013/10/edward_frenkel_on_love_and_math_what_is_it_like_to_be_a_mathematician.single.html

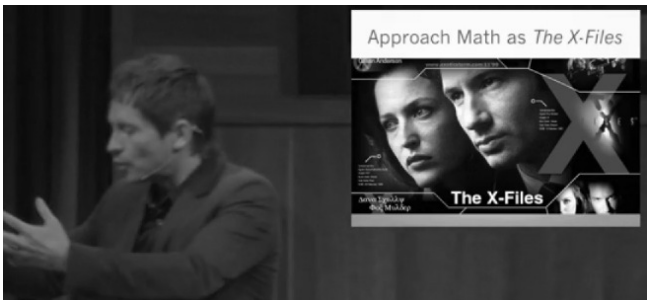
³⁴⁷ <http://thomascool.eu/Papers/ALOE/2007-08-02-ADifficultyInProofTheory.pdf>

³⁴⁸ http://en.wikipedia.org/wiki/Submission_%282004_film%29

³⁴⁹ http://en.wikipedia.org/wiki/Chris_Carter_%28screenwriter%29

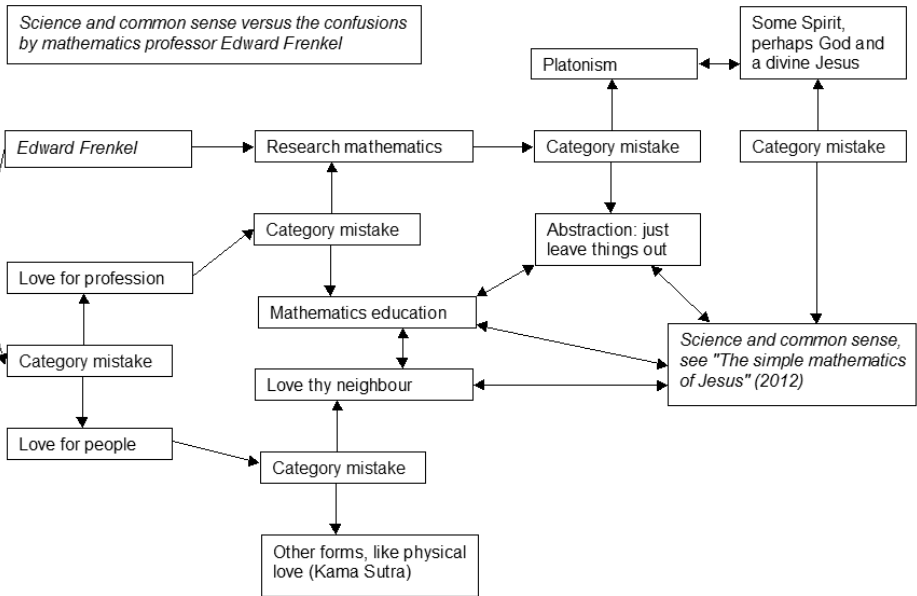
fiction. A scientist should be very careful in dealing with this. It so happens that I myself enjoy writing Science Fiction, but I use another name for doing so.³⁵⁰

I think that Frenkel crosses the line when suggesting that one should approach math as *The X-Files*. This series teases the audience from one episode to the other, and there is no meaningful message. (So it isn't good SF after a while.)



Frenkel and The X-Files (Source: LA LF video, minute 16)

A point remains that this combination of Platonism / God and “love thy neighbour” brings us to the issue of Jesus Christ. It so happens that I wrote a booklet on “*The simple mathematics of Jesus*” (SMOJ). Thus, there are all kinds of proper connections, but Edward Frenkel succeeds in confusing them.



Science and common sense versus the confusions by mathematics professor Edward Frenkel

³⁵⁰ <http://thomascool.eu/Literature/Earth/Index.html>

Edward Frenkel's denial on the economic crisis

Mathematics professor Edward Frenkel³⁵¹ is in a state of denial w.r.t. the responsibility of mathematicians for the economic crisis since August 2007. I know of only one mathematician by name who warned before the crisis developed, and that is Paul Embrechts of ETH.³⁵² Compared to him, there were various economists, read Dirk Bezemer on "*No one saw this coming*" (2009), but see also my protest w.r.t. some self-serving errors by Bezemer (including my irritation that he doesn't look into my warning, since I am still warning about more things).³⁵³

"We have to realize the power of mathematics. By now it's well-understood that the global economic crisis was caused, in part, by misuse of mathematical models.³⁵⁴ People who understood those models were actually sounding the alarm. It was the executives who had the power, who were the decision-makers, who did not understand how these formulas functioned. Their logic was: "Well, while these things work, we're making profits."" (Frenkel in *Slate* 2013)³⁵⁵

Thus, Frenkel denies and misrepresents the role of the mathematicians and "rocket scientists" who "understood those models".

A key point is that mathematicians are trained for abstraction. Thus, they are oblivious to the risks in the real world, as they are oblivious to the empirical aspects in the education in mathematics. See my book EWS (2009) that makes those points, and proves them too, with key cases from didactics and with an analysis of the political economy of the mathematics industry.

These statements by Frenkel are laudable:

"I would not tell any scientist to stop his or her research because it might have some possible evil applications. But once you discover that it does have these applications, I think it's also your responsibility to do whatever you can to prevent the discovery from being used for evil purposes. [This seems to be formulated somewhat crookedly / TC]

(...) Mathematical power is not the power of a bomb. You cannot see its effect as immediately as Hiroshima and Nagasaki. But a formula can be just as powerful in terms of controlling our lives. It can alter the course of history; it can affect millions of people.

I think we mathematicians are a little bit behind the curve. We are not fully aware of the Frankenstein that we may have already created or could create. I think that's another aspect of this responsibility of mathematicians to take a more public role to educate the public by giving them access to the beauty and power of mathematics." (Frenkel in *Slate*)

The latter, education, is precisely the answer of *Elegance with Substance* too. However, Frenkel is not aware of the conundrum: education is an empirical issue, and mathematicians are trained to think abstractly, and thus mathematicians should not be the ones to "educate the public".

³⁵¹ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

³⁵² <http://www.math.ethz.ch/~embrecht/>

³⁵³ <https://boycottholland.wordpress.com/2012/12/23/dirk-bezemer-disinforms-sweden/>

³⁵⁴ <http://www.slate.com/articles/technology/technology/2013/02/>

[should_algebra_be_in_curriculum_why_math_protects_us_from_the_unscrupulous.html](http://www.slate.com/articles/technology/technology/2013/02/should_algebra_be_in_curriculum_why_math_protects_us_from_the_unscrupulous.html)

³⁵⁵ http://www.slate.com/articles/health_and_science/new_scientist/2013/10/

[edward_frenkel_on_love_and_math_what_is_it_like_to_be_a_mathematician.single.html](http://www.slate.com/articles/health_and_science/new_scientist/2013/10/edward_frenkel_on_love_and_math_what_is_it_like_to_be_a_mathematician.single.html)

Thus professor Edward Frenkel is another deluded and abstract thinking mathematician, who is in denial of the true guilt of mathematics: (a) for the economic crisis, (b) for the sorry state of the education of mathematics, (c) for the sorry state of the education of Edward Frenkel himself.

Obviously, our deluded professor wants the mathematics industry to lift itself from the current morass, as Baron von Münchhausen so famously did. Alas, mathematicians will not be able to do so. They hold society at ransom, just to pursue their own delusions. My advice is that each nation lets its parliament investigate the issue.

(picture: Baron von Münchhausen lifting himself by his hairs from a morass, by Oskar Herrfurth (Source: wikimedia commons))

Edward Frenkel on education and inflation

2014-10-14 ³⁵⁶

Mathematics professor **Edward Frenkel** wrote an opinion “*Don’t Let Economists and Politicians Hack Your Math. Of course kids need to learn algebra*”, *Slate* Feb. 8 2013. ³⁵⁷ This appears partly a response to **Andrew Hacker**, “*Is Algebra Necessary?*”, *The New York Times*, July 28, 2012. ³⁵⁸ Frenkel also refers to **Matthew Yglesias**, “*CPI Unchained. The sneaky plan to cut Social Security and raise taxes by changing how inflation is calculated*”, *Slate* December 30 2012, ³⁵⁹ where CPI = Consumer Price index.

Let me respond as both an econometrician and teacher of mathematics.

First some facts

- In Holland, kids are allowed to graduate from middle school without algebra. The system allows for various competences. Holland isn’t perfect, though. Apparently, in the USA you either learn algebra or you drop out. If there are no alternatives then this is a rather sick approach, and then Hacker is right. See this *Journal of Humanistic Mathematics* that discusses the wider ranges of competences. ³⁶⁰
- Algebra builds upon arithmetic. Thus the source of the problems in the USA may be elementary school. See for example here. ³⁶¹
- Hacker has a nice example: “But there’s no evidence that being able to prove $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ leads to more credible political opinions or social analysis.” The example is quite nice (find the trick), and again Hacker is right that this kind of proficiency has limited value. We should have people who can do this. For others it might suffice that they know how to handle and understand a computer algebra package ³⁶² like *Mathematica* or open source Sage.
- Hacker has another nice example: “It could, for example, teach students how the Consumer Price Index ³⁶³ is computed, what is included and how each item in the index is weighted — and include discussion about which items should be included and what weights they should be given.” Below I will give an example of what would be useful to people to understand.
- The issue is *mathematics education* and not merely *mathematics*. Hacker didn’t study mathematics education but neither did Frenkel. ³⁶⁴ The opinion pieces are a bit like priests discussing marriage. People are entitled to some opinion as to how competent professionals in areas of medicine or computers should be (“deliver what you advertise”), but it is a different issue to translate such levels of competence to the educational programme to get there. It is quite possible that Hacker would be very happy with algebra in schools if the system of education would be improved

³⁵⁶ <https://boycottholland.wordpress.com/2014/10/14/edward-frenkel-on-education-and-inflation/>

³⁵⁷ http://www.slate.com/articles/technology/technology/2013/02/should_algebra_be_in_curriculum_why_math_protects_us_from_the_unscrupulous.html

³⁵⁸ http://www.nytimes.com/2012/07/29/opinion/sunday/is-algebra-necessary.html?pagewanted=all&_r=3&

³⁵⁹ http://www.slate.com/articles/business/moneybox/2012/12/chained_cpi_a_sneaky_plan_to_cut_social_security_and_raise_taxes_by_changing.html

³⁶⁰ <http://scholarship.claremont.edu/jhm/>

³⁶¹ <https://boycottholland.wordpress.com/2014/08/25/confusing-math-in-elementary-school/>

³⁶² http://en.wikipedia.org/wiki/Computer_algebra_system

³⁶³ http://topics.nytimes.com/top/reference/timestopics/subjects/c/consumer_price_index/index.html?inline=nyt-classifier

³⁶⁴ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

such that having algebra didn't cause those dropouts, and generated the overall competence that he desires. Thus we should rather not discuss algebra but the quality of mathematics education – which quality is awful.

- Frenkel, without his study of mathematics education, clearly misses the latter point, and starts defending the beauty and relevance of algebra. He is a typical mathematics professor who contributes to the dismal state of mathematics education by failing to understand that education is an empirical issue and not a playground for abstraction. Frenkel also abuses the CPI issue to scare the wits out of you, see below.

Thus, the sensible position has already been formulated in my book *“Elegance with Substance”* (2009). Blame the mathematicians and the *Slate* and *The New York Times* journalists for not paying attention.

PM. Miles Kimball & Noah Smith ³⁶⁵ remind us that the ability to do math is widespread, and that it is a myth that genes cause that there are people who can do math and people who cannot do math. They rightly point out that this myth causes needless suffering and must be gotten rid of. They refer to Hacker's article as carrying the risk of furthering that myth. I don't think that Hacker's article and proposal carry that risk. While we should work hard to improve math education and eliminate that myth, it remains that at some point in life a test will determine whether a pupil has mastered the subject or not. Rather than forcing the issue, it can be better for the pupil to let him or her proceed with the other competences and joys. Educational paths shouldn't fix life thereafter but allow for flexibility later on too.

A quantity (volume) and price index

The following method has been in use at the Dutch Central Planning Bureau (CPB) for perhaps 60 years and provides a sensible and consistent way to navigate in complex waters. It uses the Laspeyres volume and the Paasche price indices. ³⁶⁶

- Advantages of this are:
 - *Volume* and *price* changes add up to *value* changes.
 - The weights for volumes and prices are adjusted over time.
 - The choice of a different base period (for chaining) does not lead to different values for the aggregate data.
- A ('disadvantage') consequence however is that the aggregate chained volume index differs from the sum of the disaggregate chained volume indices. This however cannot be seen as a 'disadvantage' since there is no reason why that sum should hold.

Consider a fictitious example of indices for the national old age pension (social security) for the age 65+. Suppose that the pension allows for (1) bread, (2) coffee and (3) some health insurance. This is called the 'basket'. Then we calculate: (4) the aggregate volume and price indices. The outlays change from year to year, partly because of entrants and departures, and partly because of changing tastes and reactions to price changes.

We will use these symbols for a fictitious change from 2013 to 2014:

³⁶⁵ <http://qz.com/139453/theres-one-key-difference-between-kids-who-excel-at-math-and-those-who-dont/>

³⁶⁶ http://en.wikipedia.org/wiki/Price_index

<i>Symbols used in this discussion</i>	pd = price change in level
vd = volume change in level	pp = price change in percentage of vn
vp = volume change in percentage	wn = value in level, current prices
vn = volume level (value in former prices)	wn - 1 = lagged value in level

Price, volume and value, symbols for level and absolute and relative changes

Let us assume that an average pensioner eats 100 loaves of bread per annum, at EUR 1 per loaf (wholesale price) and that this quantity changes by 5%. Let the price remain constant. This gives us the **first line** of the table below. Expenditure (wn) changes from EUR 100 to 105.

In the **second line** we record that 450 cups of coffee were consumed in 2013 at EUR 1.3 per cup, thus at a total expenditure (wn) of EUR 585. Due to a VAT change affecting coffee only, the price rises by 7.7% to EUR 1.5 and hence we see a drop in consumption of 4.4% to 430 cups of coffee per year. Expenditure on this item changes from EUR 585 to EUR 602.

In the **third line** we assume that the pensioner has medical attention for one hour per week, thus 52 hours per annum, at the cost of EUR 200 per hour. Thus the expenditure in 2013 is EUR 10400. In 2014 due to aging, the average pensioner requires 53 hours per year, and the medical cost rises to EUR 210 per hour (5%). The outlays from medical insurance rises to EUR 11130 per annum.

Collecting these data in the **fourth line**, the total expenditure per pensioner is EUR 11085 in 2013 and rises to EUR 11837 in 2014.

- The 'aggregate volume index' (2013 = 100) is 101.6%.
- The 'aggregate price index' is 105.1%.
- The 'aggregate value index' can be found by properly multiplying these, and is consistent with calculating $11837 / 11085 * 100$.

	wn-1	vd	vp	vn	pd	pp	wn
1	100.0	5.0	5.0	105.0	0.0	0.0	105.0
2	585.0	-26.0	-4.4	559.0	43.0	7.7	602.0
3	10400.0	200.0	1.9	10600.0	530.0	5.0	11130.0
4	11085.0	179.0	1.6	11264.0	573.0	5.1	11837.0

Line 4 gives the aggregate of lines 1 – 3, in value, volume and price

(Calculated with `c = ChainIndexTable[{{100., 105.}, {450., 430.}, {52., 53.}}, {{1., 1.}, {1.30, 1.40}, {200., 210.}}]` in *The Economics Pack*.³⁶⁷)

The method uses only arithmetic but the highschool student has to understand quite a lot of concepts: value, volume, price, level, absolute and percentage change, averaging, using different weights. The student can show insight in that the figures make some sense (a price rise causes a quantity drop), and that the aggregate outcome is dominated by the large component. The student will also see that when more of such calculations are combined (say pensioners, families with and without children, singles; with subaggregates in the provinces or states) then the system remains consistent, so that the method of aggregation does not introduce curious distortions.

³⁶⁷ <http://thomascool.eu/TheEconomicsPack/index.html>

The aggregate volume and price changes for baskets of single years can be 'chained' by multiplying the annual results. The aggregate price index in 2014 with base (2010 = 100) would be: $100 * (1 + pp[2011] / 100) * \dots * (1 + pp[2014] / 100)$. The advantage of this chaining is that the basket is not fixed but follows actual expenditure.

A criticism is: since above VAT rise reduces real spending power, then there is more weight upon less real consumption. The above allows pensioners a rise of expenditure from EUR 11085 to EUR 11837, but 'adequate compensation' should require EUR 26 more per person because of their induced loss of their cups of coffee. However, is this statistics or politics? In terms of policy, giving EUR 26 more does not guarantee that this is spent on coffee. In terms of statistics, there is a choice of using weights of the baskets of either 2013 or 2014. The method of using Layspeyers volumes and Paasche prices allows to calculate the result for 2014 using the weights from 2014.

The role of statisticians is to also inform policy makers about the observed changes in the basket. The choice of the basket is politics.

Thus, one would agree with Hacker that it would be useful that this method would be taught in highschool in Holland, so that students know what their CPB is doing. Similarly for the USA. Yglesias refers to Jill Leyland (2011)³⁶⁸ for the UK.

In Holland, Bert Balk³⁶⁹ has pointed to the properties of the Divisia – Törnqvist index³⁷⁰ but hasn't convinced others, or me, yet that its modestly larger computational complexity generates better results (and other criteria to select than the above). It is getting to be used more often in productivity measurement, but note that hours worked and wage inflation can also be aggregated in above manner by distinguishing levels of productivity and associated wages (and it is curious that this often isn't done).

Frenkel's embrace of conspiracy theory

The issue is also discussed in this Wikipedia article³⁷¹ that at the time of writing this has not yet been sabotaged by MIT students and their formulas. Yglesias referred to above observes:

"(...) the point is that there's no unique right or wrong answer (...) for how to treat product shifting, and its impact on individuals' welfare will vary enormously. This matters a great deal for Social Security, however, because benefit levels are adjusted upward each year in line with inflation. If Congress decides that chained index is the "right" measure of inflation, benefit levels will be lower than currently predicted and the deficit will go down."

The answer is of course *that social security should not be indexed on only inflation but on the general rise in welfare*. (In the above: on wn. This should also hold for tax brackets.) If there is a rise in productivity, then let also the elderly benefit from it. If everyone has a computer then let also senior citizens be allowed to get one. Thus, we have an easy issue of policy making here.

This is also what Yglesias concludes:

"The central point is that it's politics all the way down. (...) glib talk about apples and "better" inflation calculations masks a policy that if done sloppily could be quite damaging to the low-income elderly."

³⁶⁸ <http://www.significancemagazine.org/details/webexclusive/1314363/RPI-versus-CPI---The-Definitive-Account.html>

³⁶⁹ <http://www.rsm.nl/people/bert-balk/publications/>

³⁷⁰ http://en.wikipedia.org/wiki/T%C3%B6rnqvist_index

³⁷¹ http://en.wikipedia.org/wiki/United_States_Chained_Consumer_Price_Index

However, mathematics professor Edward Frenkel claims that you can only understand the issue when you have learned algebra, while preferably you should also know *gauge theory*. In his opinion (his is an opinion piece) there is also a political conspiracy, supported by economists, to exploit your lack of understanding of mathematics, and keeping you there.

(1) Frenkel's major claim:

"Is economics being used as science or as after-the-fact justification, much like economic statistics were manipulated in the Soviet Union? More importantly, is anyone paying attention? Are we willing to give government agents a free hand to keep changing this all-important formula [for CPI] whenever it suits their political needs, simply because they think we won't get the math?"

(...) What seems to be completely lost on Hacker and authors of similar proposals is that the calculation of the CPI, as well as other evidence-based statistics, is in fact a difficult mathematical problem, which requires deep knowledge of all major branches of mathematics including ... advanced algebra. Whether we like it or not, calculating CPI necessarily involves some abstract, arcane body of math. [Frenkel doesn't seem to be aware of above practical solution.]

(...) The inflation index must account for this, so we have to find a way to compare the baskets today and a year ago. This turns out to be a hard mathematical problem that has perplexed economists for more than a century and still hasn't been completely solved. But even to begin talking about this problem, we need a language that would enable us to operate with symbolic quantities representing baskets and prices—and that's the language of algebra! [Frenkel confuses above simple arithmetic and algebra with the somewhat more involved algebra that Hacker was speaking about.]

(...) As Weatherall explains in his book, to implement a true cost-of-living index, one actually has to use the so-called "gauge theory." This mathematics is at the foundation of a unified physical theory of three forces of nature: electromagnetism, the strong nuclear force, and the weak nuclear force.³⁷² (Many Nobel Prizes have been awarded for the development of this unified theory; it was also used to predict the Higgs boson, the elusive elementary particle recently discovered at the Large Hadron Collider under Geneva.)³⁷³

(...) So that's where we find ourselves today: Politicians are still eager to exploit backdoor mathematical formulas for their political needs, economists are still willing to play along, and no one seems to care about finding a scientifically sound solution to the inflation index problem using adequate mathematics. And the public—well, very few people are paying attention. And if we follow Hacker's prescriptions and further dumb down our math education, there won't be anyone left to understand what's happening behind closed doors." [Well, there is a huge economic literature on indexation.]

(2) Frenkel alarms us – a call to arms – shifting the blame to politicians and economists instead seeing his own fault:

"Now is the time *not* to reduce math curriculum at schools, but to *expand* it, taking advantage of new tools in education: computers, iPads, the wider dissemination of knowledge through the Internet. Kids become computer literate

³⁷² <http://blogs.discovermagazine.com/cosmicvariance/2012/11/22/thanksgiving-7/>

³⁷³ http://www.slate.com/articles/technology/future_tense/2012/07/higgs_boson_announcement_from_cern_why_the_god_particle_is_so_important_.html

much earlier these days, and they can now learn mathematical concepts faster and more efficiently than any previous generation. But they have to be pointed in the right direction by teachers who inspire them to think big. This can only be achieved if math is not treated as a chore and teachers are not forced to spend countless hours in preparation for standardized tests. Math professionals also have a role to play: Schools should invite them to help teachers unlock the infinite possibilities of mathematics to students, to show how a mathematical formula can be useful in the real world and also be elegant and beautiful, like a painting, a poem, or a piece of music.”

It is hard to see a political conspiracy to block the education of mathematics. Kids and students already spent an amazing number of hours on arithmetic and other mathematics. It are the mathematicians and the abstractly trained teachers of mathematics who are the spoil-sports. They do not see that education is an empirical issue, and they insert their hobbies and traditions into their allotted time, without wondering whether they shouldn't pay attention to their students. As said, Frenkel hasn't studied education,³⁷⁴ so he is in breach of the integrity of science, claiming expertise that he doesn't have.

(3) Frenkel claims a conflict of competence / interest for Hacker:

“In his book, Weatherall made an admirable effort to start a serious conversation about the need for a new mathematical theory of the CPI. But guess who reviewed this book in the *New York Review of Books*?³⁷⁵ Andrew “we don't need no algebra” Hacker! There is nothing wrong with healthy debate; it can only be encouraged. But something is wrong when an opinionated individual who has demonstrated total ignorance of a subject matter gets called on over and over again as an expert on that subject.”

I haven't read that review yet, but it seems to me that Frenkel wrongly disqualifies Hacker. The label “Hack” in Frenkel's opinion piece would be like someone to write about “A mathematical proof of conspiracy by Dr. Frenkelstein”.

(4) Frenkel claims that there was already unscientific manipulation in the Boskin commission 1996:³⁷⁶

“But what most people don't realize is that something similar had already happened in the past. A new book, *The Physics of Wall Street* by James Weatherall,³⁷⁷ tells that story: In 1996, five economists, known as the Boskin Commission, were tasked with saving the government \$1 trillion. They observed that if the CPI were lowered by 1.1 percent, then a \$1 trillion could indeed be saved over the coming decade. So what did they do? They proposed a way to alter the formula that would lower the CPI by *exactly* that amount!”

“(…) The fact that gauge theory also underlies economics was a groundbreaking discovery³⁷⁸ made by the economist Pia Malaney and mathematical physicist Eric Weinstein around the time of the Boskin Commission. Malaney, who was at the time an economics Ph.D. student at Harvard,³⁷⁹ tried to convey the

³⁷⁴ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

³⁷⁵ <http://www.nybooks.com/articles/archives/2013/jan/10/how-he-got-it-right/?pagination=false>

³⁷⁶ http://en.wikipedia.org/wiki/Boskin_Commission

³⁷⁷

http://www.amazon.com/gp/product/0547317271/ref=as_li_ss_tl?ie=UTF8&camp=1789&creative=390957&creativeASIN=0547317271&linkCode=as2&tag=slatmaga-20

³⁷⁸ <http://pirsa.org/06050010/>

³⁷⁹

<http://lists.perimeterinstitute.ca/pipermail/piuwcomplex/attachments/20090205/7f2cccd3/attachment-0002.pdf>

importance of this theory for the index problem to the Harvard professor Dale Jorgenson, one of the members of the Boskin Commission, but to no avail. In fact, Jorgenson responded by throwing her out of his office. Only recently, George Soros' *Institute for New Economic Thinking* finally gave Malaney and Weinstein long overdue recognition and is supporting their research.³⁸⁰ But their work still remains largely ignored by economists."

Mark Thoma had some comments on this in 2006³⁸¹ and quotes BLS in 2008.³⁸² I am not tempted to follow this up here. I suppose that there is no conflict of interest in Frenkel and Weinstein appearing jointly at the Speyer Legacy school,³⁸³ both apparently without a background in the education in mathematics.

Overall, I am happy to include this subject in a parliamentary enquiry on unemployment and/or another such enquiry on the dismal state of mathematics education.

³⁸⁰ <http://ineteconomics.org/grants/geometric-marginalism>

³⁸¹ http://economistsview.typepad.com/economistsview/2006/03/was_the_calcula.html

³⁸² <http://economistsview.typepad.com/economistsview/2008/09/common-misconce.html>

³⁸³ <http://www.speyerlegacyschool.org/page.cfm?p=442&newsid=35>

Edward Frenkel on democracy

2014-10-17 ³⁸⁴

In the 9-minute Numberphile interview *Why do people hate mathematics?* ³⁸⁵ – see yesterday's discussion ³⁸⁶ – professor of mathematics Edward Frenkel states, in minute three:

“Georg Cantor said: *“The essence of mathematics lies in its freedom.”* But I would like to augment this with the following: *Where there is no mathematics there is no freedom.* So mathematics is essential to our freedom, to the functioning of our democracy. (...) Our ignorance can be misused by the powers that be. And for us ... as citizens in this Brave New World ... we have to be more aware of mathematics, we have to know and appreciate its power – to do good but also to do ill.” (Edward Frenkel, Jan 19 2014)

We can only applaud this. In my *Elegance with Substance* (EWS)(2009):

“Mathematics is a great liberating force. No dictator forces you to accept the truth of the Pythagorean Theorem. You are free to check it for yourself. You may even object to its assumptions and invent non-Euclidean geometry. Mathematical reasoning is all about ideas and deductions and about how far your free mind will get you – which is amazingly far. But you have to be aware of reality if you say something about reality.” (EWS p9)

“Democracy is an important concept. The mathematics of voting is somewhat complex. It would be beneficial for society when its citizens understand more about the mathematics behind election results. Students in the USA have a Government class where such aspects can be indicated. Political Science as a subject has not reached highschool in general. Much can be said in favour of including the subject in economics, since the aggregation of preferences into a social welfare function is a topic of Political Economy. See page 59 and Colignatus (2007b) *Voting theory for democracy* (VTFD) for details and other references. Most economists will be unfamiliar with the topic and its mathematics though and thus it may well be practical to include it in the mathematics programme.” (EWS p48)

However, let us also look at key criticism:

1. Mathematician Kenneth Arrow presented his 'impossibility theorem' in his 1951 thesis. It holds, in his own words: *“there is no social choice mechanism which satisfies a number of reasonable conditions”* Palgrave (1988:125) and quoted in *Voting Theory for Democracy* (VTFD)(2014) 4th edition p240. Thus collective choice would require us to be unreasonable. Mathematician Arrow continued in economics and got the Nobel Prize in economics for this and other work.
2. Mathematicians, political scientists and economists have tried since 1950 to debunk Arrow's result, but did not find real solutions. These areas of science have become a force against democracy. Collective choice would require us to be unreasonable, and this would be scientifically proven.
3. When I showed in 1990 that Arrow's words do not fit his mathematics, and a bit later that his result was either inconsistent or incomplete, hell broke out. My paper was suppressed from discussion and publication. A mathematician who

³⁸⁴ <https://boycottholland.wordpress.com/2014/10/17/edward-frenkel-on-democracy/>

³⁸⁵ <https://www.youtube.com/watch?v=Yexc19j3TjE>

³⁸⁶ <https://boycottholland.wordpress.com/2014/10/16/why-do-people-hate-mathematics-qqq/>

was supposed to review VTFD (3rd edition) started slandering. See the journal *Voting Matters* (April 2013).³⁸⁷ See my point however that there is a distinction between 'voting' (counting ballots) and 'deciding'.³⁸⁸ And see VTFD for the more involved presentation (starting with matricola).

4. It has been impossible to find someone in Holland to discuss this issue rationally. Here is a report in English on a working group in social choice theory.³⁸⁹ Here is a page in Dutch.³⁹⁰ On a website for highschool students, Kennislink.nl,³⁹¹ deluded mathematician Vincent van der Noort, who did not properly study the issue, claims that "*democracy isn't entirely fair*", thus encouraging highschool students to use their elbows. The editors refuse to correct this falsehood and selective use of sources (or mystery, since Vincent doesn't define fairness).

I suppose that professor Frenkel discusses democracy in general, without thinking specifically about Arrow's 'Theorem'. Perhaps he doesn't know about it, and would be surprised that it would be 'mathematically proven' that some degree of dictatorship would be necessary. However, to some extent we can agree with him. Good education in mathematics will do wonders for liberty and democracy. But, my point again: the definition of 'good education in mathematics' is subtle. See these quotes from EWS too:

"With respect to logic and democracy, Colignatus (2007ab, 2008b), updated from 1981 / 1990, considers statements by mathematicians that have been accepted throughout academia and subsequently society on the basis of mathematical authority. It appears however that those statements mix up true mathematical results with interpretations about reality. When these interpretations are modelled mathematically, the statements reduce to falsehoods. Society has been awfully off-track on basic notions of logic, civic discourse and democracy. Even in 2007, mathematicians working on voting theory wrote a Letter to the governments of the EU member states advising the use of the Penrose Square Root Weights (PSRW) for the EU Council of Ministers. See Colignatus (2007c) on their statistical inadequacy and their misrepresentation of both morality and reality.

Over the millennia a tradition and culture of mathematics has grown that conditions mathematicians to, well, what mathematicians do. Which is not empirical analysis. Psychology will play a role too in the filtering out of those students who will later become mathematicians. After graduation, mathematicians either have a tenure track in (pure) mathematics or they are absorbed into other fields such as physics, economics or psychology. They tend to take along their basic training and then try to become empirical scientists.

The result is comparable to what happens when mathematicians become educators in mathematics. They succeed easily in replicating the conditioning and in the filtering out of new recruits who adapt to the treatment. For other pupils it is hard pounding." (EWS p10)

PM. See where Georg Cantor went wrong: *Contra Cantor Pro Occam* (2012, 2013)³⁹² - now superseded by FMNAI (2015).

³⁸⁷ <http://www.votingmatters.org.uk/ISSUE30/INDEX.HTM>

³⁸⁸ <http://econpapers.repec.org/paper/pramprapa/34919.htm>

³⁸⁹ <http://thomascool.eu/Thomas/English/Science/Letters/SCT-working-group.html>

³⁹⁰ <http://thomascool.eu/Thomas/Nederlands/Wetenschap/Artikelen/2013-02-14-PasOpMetWiskundeOverVerkiezingen.html>

³⁹¹ <http://www.kennislink.nl/publicaties/is-democratie-wiskundig-onmogelijk>

³⁹² <http://thomascool.eu/Papers/Aloe/2012-03-26-CCPO-PCWA.pdf> but see FMNAI (2015)

The math industry of confusing people

October 15 2014 ³⁹³

There is a curious argument that $1 + 2 + 3 + 4 + \dots = -1/12$ (*New York Times* February 3 2014). ³⁹⁴

Some pronounce this as “minus one over twelve” but this weblog proposes “min per ten-two” ³⁹⁵ or “negative per ten-two”. On occasion we employ $H = -1$, to be pronounced as “eta”. ³⁹⁶ Thus “eta per ten-two” is okay as well. We can also use $1/12 = 12^H$, pronounced as “per ten-two”. (The Germans would pronounce H as “Ha” and we would not want them to be laughing all the time.)

The *NY Times* article and *Numberphile* video was debunked by other mathematicians and physicists on the internet, see some links below. However, this weblog looks at issues from the angles of both econometrics and the education of mathematics. From these angles we find:

1. The article and video do not satisfy the conditions of didactics.
2. There appears to be a large mathematical industry to confuse people.

Mathematics professor Edward Frenkel is part of the m  le  . ³⁹⁷ He is quoted in above article (and can be heard in some video’s saying similar things):

“This calculation is one of the best-kept secrets in math.” “No one on the outside knows about it.”

The article states:

“In modern terms, Dr. Frenkel explained, the gist of the calculations can be interpreted as saying that the infinite sum has three separate parts: one of which blows up when you go to infinity, one of which goes to zero, and minus $1/12$. The infinite term, he said, just gets thrown away.”

The latter is rather curious. Why are you allowed to throw infinity away ? If you take something from infinity before you throw infinity away, why would you select -12^H and not something else ?

Let us consider the situation, and start with Grandi’s Series. Personally, I was reminded about an approximation to -12^H found last year, but since it is only an approximation this comment has been put into **Appendix A**.

An unwarranted deduction

In *Numberphile*, *Thomson’s Lamp*, there is this video discussion ³⁹⁸ about “Grandi’s Series” G . ³⁹⁹ That discussion (and on wikipedia retrieved today, see **Appendix B**) is unwarranted. The proper deduction is:

$$G = 1 - 1 + 1 - 1 + \dots = (1 + 1 + 1 + \dots) - (1 + 1 + 1 + \dots) = \infty - \infty = \text{undefined}$$

³⁹³ <https://boycottholland.wordpress.com/2014/10/15/the-math-industry-of-confusing-people/>

³⁹⁴ http://www.nytimes.com/2014/02/04/science/in-the-end-it-all-adds-up-to.html?_r=1

³⁹⁵ <http://thomascool.eu/Papers/Math/2012-04-16-NiceNumbers.pdf>

³⁹⁶ <https://boycottholland.wordpress.com/2014/08/30/taking-a-loss/>

³⁹⁷ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkel-didnt-study-math-education/>

³⁹⁸ http://youtu.be/PCu_BNNI5x4

³⁹⁹ http://en.wikipedia.org/wiki/Grandi%27s_series

It is an altogether different question that we can look at the *average of the series of partial sums*. The *Lamp* mentions this (to their credit) but uses the same plus-sign which is unwarranted. We should use a different plus sign. Then we find:

$$G' = 1 \oplus H \oplus 1 \oplus H \oplus \dots = 1 - G' \quad \text{so that} \quad G' = 2^H$$

Partial sums of G : 1, 0, 1, 0, 1, 0,

Summing (again !) those into a series: $1 + 0 + 1 + 0 + 1 + \dots$

Averaged series G' : $1 / 1$, $(1 + 0) / 2 = 2^H$, $(1 + 0 + 1) / 3 = 2/3$, $2 / 4 = 2^H$, ...

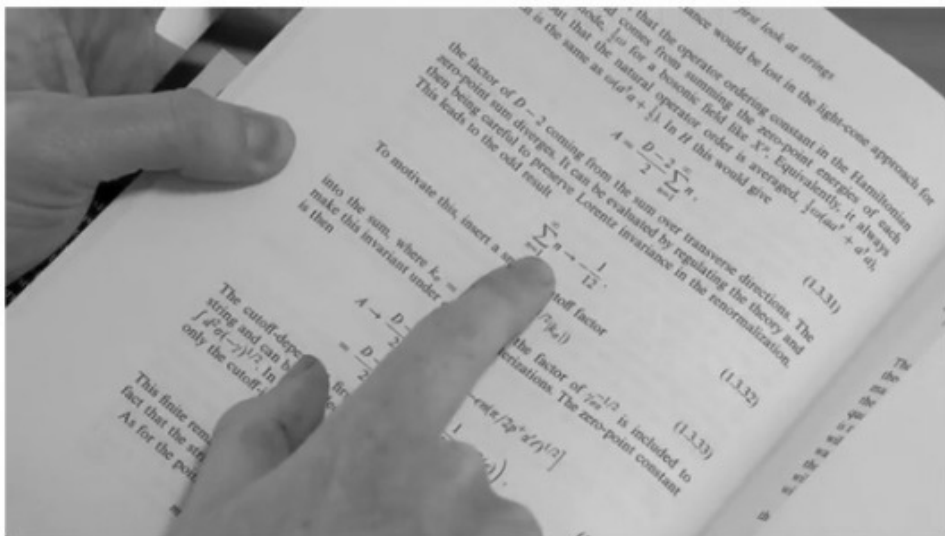
The mystery completely disappears.

Divergent series can be operated upon, with differences, sums, averages, until you find something that converges. You might use this to catalog them.

That *Lamp* video discusses turning on and off an actual lamp, in ever smaller fractions $2^{-(n)}$ of a minute, starting at zero, such that the process should stop after two minutes (we can calculate that period mathematically): and then the question is whether the lamp is on or off. This is a badly defined problem. It is the same as the Zeno paradox of Achilles and the hare. A mathematical story using terms from physics doesn't make it proper physics.

A string theory mystery

I am no physicist and know nothing about string theory, but am a bit perplexed when this other *Numberphile* video shows that page 22 for $1 + 2 + 3 + 4 + \dots \Rightarrow -1 / 12$. Note the *arrow* rather than the *equality sign*. It remains a question: are they really taking the limit ? Hopefully the deduction in string theory is more to the point than the deduction given in the video. The deduction in that video clearly is not sound. It uses $G = 2^H$ but we have shown that only $G' = 2^H$. Indeed, see below for some links to physics websites that show that the video is crooked.



ASTOUNDING: $1 + 2 + 3 + 4 + 5 + \dots = -1/12$ Video by Numberphile

Page 22 of Joseph Polchinski, "String Theory" (Source: Numberphile video)

The *Numberphile* video uses three series. Confusingly it uses the normal plus sign but let us consider the idea that these would concern *averages of a series of partial sums* (with \oplus instead of $+$). Series $S_1 = G$ and S_2 is another form of $\infty - \infty = \text{undefined}$.

$$S_1 = 1 - 1 + 1 - 1 + 1 - 1 + 1 - \dots$$

$$S_2 = 1 - 2 + 3 - 4 + 5 - 6 + 7 - \dots$$

$$S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots$$

ASTOUNDING: $1 + 2 + 3 + 4 + 5 + \dots = -1/12$ Video by Numberphile

Read (+) and (+ H) instead of plus and minus (Source: Numberphile video)

Let us repeat above procedure for S . Since there are no negative values involved, the series merely explodes, and obviously the outcome cannot be negative.

$$S' = 1 \oplus 2 \oplus 3 \oplus 4 \oplus \dots$$

Partial sums of S : 1, 3, 6, 10, 15,

Summing (again !) those into a series: $1 + 3 + 6 + 10 + 15 + \dots$

$$\text{Averaged series } S': 1 / 1, (1 + 3) 2^H = 2, 10 3^H = 3 + 3^H, 16 4^H = 4 \dots$$

The *Numberphile* team has a longer video⁴⁰⁰ on the sum of the natural numbers that uses the Euler-Riemann Zeta⁴⁰¹ function to argue their point, supposedly in 'proper fashion'. However, they do not discuss the paradoxes here, and thus leave the reader confused. For example, they also refer to the basic geometric series,⁴⁰² differentiate this, and then substitute $r = -1$ to create S_2 (calling this 'analytic continuation'), but, if the original geometric series is undefined for $r = -1$ (and then actually generates the Grandi Series again): why do you think that you can do this ?

⁴⁰⁰ https://www.youtube.com/watch?annotation_id=annotation_2226441133&feature=iv&src_vid=w-l6XTVZXww&v=E-d9mgo8FGk

⁴⁰¹ http://en.wikipedia.org/wiki/Riemann_zeta_function

⁴⁰² http://en.wikipedia.org/wiki/Geometric_series

Let $s = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$.

Then $rs = ar + ar^2 + ar^3 + ar^4 + \dots + ar^n$

Then $s - rs = a - ar^n$

Then $s(1 - r) = a(1 - r^n)$, so $s = a \frac{1 - r^n}{1 - r}$ (if $r \neq 1$).

As n goes to infinity, the absolute value of r must be less than one for the series to converge. The sum then becomes

$$a + ar + ar^2 + ar^3 + ar^4 + \dots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r} \Leftrightarrow |r| < 1$$

When $a = 1$, this simplifies to:

$$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r},$$

Geometric series converges for $-1 < r < 1$ (Source: wikipedia)

See some physics links

The **1 + 2 + ... = -1/12** video got 1.5 million hits and a fair amount of reactions from physicists. Their point is that Riemann and they are doing their job. See Steven Corneliussen in *Physics Today*⁴⁰³ and Phil Plait at *Slate*,⁴⁰⁴ for example. Plait has this quote from Jordan Ellenberg:

"It's not quite right to describe what the video does as "proving" that $1 + 2 + 3 + 4 + \dots = -1/12$. When we ask "what is the value of the infinite sum," we've made a mistake before we even answer! Infinite sums don't have values until we assign them a value, and there are different protocols for doing that. We should be asking not what IS the value, but what should we define the value to be? There are different protocols, each with their own strengths and weaknesses. The protocol you learn in calculus class, involving limits, would decline to assign any value at all to the sum in the video. A different protocol assigns it the value $-1/12$. Neither answer is more correct than the other."

This is not entirely correct. Once you have defined "addition" and "equals" then you are stuck with it. Yes, you are free to find another protocol, but, beware of using "addition" and "equals" in general publications and education in another sense than people understand, because then you create confusion.

It seems to me that *Physics Buzz* is the most enlightening on what the real intention is.⁴⁰⁵

Some nice quotes however

However, to soften our conclusion, the *NY Times* article by Dennis Overbye provides some nice quotes:

"The problem with infinity is that you can't stop. You never get there. It's more of a journey than a destination.

⁴⁰³ <http://scitation.aip.org/content/aip/magazine/physicstoday/news/10.1063/PT.5.8029>

⁴⁰⁴ http://www.slate.com/blogs/bad_astronomy/2014/01/18/follow_up_the_infinite_series_and_the_mind_blowing_result.html

⁴⁰⁵ <http://physicsbuzz.physicscentral.com/2014/01/redux-does-1234-112-absolutely-not.html>

Niels Henrik Abel, whose notion of an Abel sum plays a role here, once wrote, "The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever."⁴⁰⁶

Of course there is also Wigner again:

"To him and others, this is just another example of what the eminent physicist Eugene Wigner called the "unreasonable effectiveness of mathematics."⁴⁰⁷ Why should such woolly and abstract concepts as zeta functions or imaginary numbers, the products of a chess game in our minds, have such relevance in describing the world?"

However, as mathematics = abstraction, and abstraction = leaving out aspects, it should not be surprising that if you start with the world and leave things out then you still have something. See here for complex numbers,⁴⁰⁸ and check the steps in turning around a circle:

1, i , $H = i^2$, $H i$, 1
(start at 1 = {1, 0}, quarter turn, half turn, three-quarters turn, back to 1)

Appendix A: $\phi^2 / \Theta \approx 5 / 12$

Remember that we found that $\phi^2 / \Theta \approx 5 / 12$ with an error of 6 per million,⁴⁰⁹ where $\phi = 1.618033989...$ (pronounce 'phi') is the golden ratio, and where $\Theta = 2 \pi = 6.283185307....$ (pronounce 'archi').

$$\phi^2 / \Theta = 0.416673050492137...$$

$$5 / 12 = 0.4166666...$$

$$\phi^2 / \Theta - 5 / 12 = 0.00000638382547060161...$$

$$\phi^2 / \Theta - 2^H \approx -12^H \quad (\text{with the same error})$$

Thus, the suggestion is that when some physics formula generates the number -12^H , look whether this kind of thing might be involved. We came upon this from an application. The relation holds by approximation only, however, and might be abused again to confuse people.

⁴⁰⁶ <http://www.britannica.com/EBchecked/topic/1025/Niels-Henrik-Abel>

⁴⁰⁷ <http://www.nytimes.com/1995/01/04/obituaries/eugene-wigner-92-quantum-theorist-who-helped-usher-in-atomic-age-dies.html>

⁴⁰⁸ <https://boycottholland.wordpress.com/2014/10/15/the-danger-of-complex-number-i/>

⁴⁰⁹ <https://boycottholland.wordpress.com/2013/01/27/pyramids-and-the-meter/>

Appendix B: Wikipedia 2014-10-15 on Grandi's series contributes to confusion

Heuristics [\[edit\]](#)

One obvious method to attack the series

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \dots$$

is to treat it like a telescoping series and perform the subtractions in place:

$$(1 - 1) + (1 - 1) + (1 - 1) + \dots = 0 + 0 + 0 + \dots = 0.$$

On the other hand, a similar bracketing procedure leads to the apparently contradictory result

$$1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots = 1 + 0 + 0 + 0 + \dots = 1.$$

Thus, by applying parentheses to Grandi's series in different ways, one can obtain either 0 or 1 as a "value". (Variations of this idea, called the Eilenberg–Mazur swindle, are sometimes used in knot theory and algebra.)

Treating Grandi's series as a divergent geometric series we may use the same algebraic methods that evaluate convergent geometric series to obtain a third value:

$$S = 1 - 1 + 1 - 1 + \dots, \text{ so}$$

$$1 - S = 1 - (1 - 1 + 1 - 1 + \dots) = 1 - 1 + 1 - 1 + \dots = S,$$

resulting in $S = 1/2$. The same conclusion results from calculating $-S$, subtracting the result from S , and solving $2S = 1$.^[1]

The above manipulations do not consider what the sum of a series actually means. Still, to the extent that it is important to be able to bracket series at will, and that it is more important to be able to perform arithmetic with them, one can arrive at two conclusions:

- The series $1 - 1 + 1 - 1 + \dots$ has no sum.^{[1][2]}
- ...but its sum *should* be $1/2$.^[2]

In fact, both of these statements can be made precise and formally proven, but only using well-defined mathematical concepts that arose in the 19th century. After the late 17th-century introduction of calculus in Europe, but before the advent of modern rigor, the tension between these answers fueled what has been characterized as an "endless" and "violent" dispute between mathematicians.^{[3][4]}

Wikipedia's discussion October 15 2014 on Grandi's series (Source: wikipedia)

Archives

Replace arXiv by viXra & PressForward

2014-10-09 ⁴¹⁰

Yesterday the moderators at arXiv.org informed me that my submission of *Education, division & derivative: Putting a Sky above a Field or a Meadow* ⁴¹¹ at math.HO was rejected and removed. The situation is quite annoying. **The attitude at arXiv.org is unscientific and uncivilised.** When people do not state a developed argument then you are left with guessing. And if they would give a reason then you would hope that they would listen to a reply, but this isn't their attitude. **They just delete, no questions asked.**

It is only an archive. In my idea it should suffice for a research archive that you have a university degree, which by definition ought to prepare you for science. If universities are not up to that task then do something about universities but do not try to second-guess them.

After some deliberation my suggestion to the world is to use other archives like viXra.org in combination with *PressForward – the WordPress Plugin*. ⁴¹² Leave the hassle of arXiv and concentrate on *PressForward*.

To: thomas cool / thomas colignatus

From: arXiv Moderation

Subject: [moderation #129703] arXiv:submit/1073199 removed

Date: Wed, 8 Oct 2014

Your submission has been removed upon a notice from our moderators, who determined it to be inappropriate for arXiv. Please find another forum.

Please do not resubmit this paper without contacting moderation and obtaining a positive response. Resubmission of removed papers may result in suspension of submission privileges.

For more information on our moderation policies see:

<http://arxiv.org/help/moderation>

— arXiv moderation

What is *wrong* with arXiv.org ? Why do they act like this ?

In 2009 they rejected a crosslink of *Elegance with Substance* (EWS) (AMS Book List, Notices Vol 58, No 11, p1474) ⁴¹³ too. That book / PDF is also at the economics MPRA in Munich since it also deals with the political economy of the mathematics industry, but EWS discusses mathematics and its education and points of didactics, and thus ought to warrant a (cross) listing in arXiv.

⁴¹⁰ <https://boycottholland.wordpress.com/2014/10/09/replace-arxiv-by-vixra-and-pressforward/>

⁴¹¹ <http://thomascoll.eu/Papers/Math/2014-09-08-Sky-Field-Meadow.pdf>

⁴¹² <http://pressforward.org/announcing-the-pressforward-plugin/>

⁴¹³ <http://www.ams.org/notices/201110/rtx111001471p.pdf>

The issue is not merely the use of formula's and theorems and proofs. Check "*Through a Glass Darkly*"⁴¹⁴ by Steven Krantz, 2008, and try to spot the theorem.

Part of the problem is that Research Mathematics (RM) is not aware that Mathematics Education (ME) a.k.a. School Mathematics (SM) is an empirical issue. ME research (MER) is an empirical science which is a different ballgame than RM are used to. There is no specific MER category so you are left with math.HO. Some papers that partly deal with education apparently are accepted. A search on mathematics AND education gave 382 hits (see one of mine ⁴¹⁵). But it is obvious that the mathematics education research community is not very active on arXiv.org. (There will be more reasons, see PM 4 below.)

The problem at arXiv.org is: they are horribly afraid of cranks. The Einsteins of the world are afraid of cranks, can you imagine ? The fear of the pest, cholera, leprosy, AIDS and perhaps now also ebola is in them. So, okay, they are not real Einsteins, but high priests of an academy turned into a bureaucracy. They hide under the cloak of a library, perverting what libraries are intended for.

Phil Gibbs created viXra.org ⁴¹⁶ as an alternative for time-stamping and getting a fixed URL (FURL). It is said that only 15% of that archived material gets adopted in the peer reviewed journals. Of course peer review is no real check on quality yet. It all depends what the final readership will think. Smart readers will look at more material than just appears in the official journals – material such as in arXiv.

The very use of arXiv should make the moderators modest about the importance of peer review but tends to make them arrogant as the accountants of truth. People at arXiv.org will be very happy with viXra.org as it saves them a lot of work. Perhaps I am not paranoid enough and should think that Gibbs merely is a front of arXiv to get rid of what they consider the burden of noise. I am amazed to see an author submitting the words of Jesus as quoted to him. viXra has been called a 'crank magnet'.⁴¹⁷

Overall, Gibbs has a good point on that FURL. Researchers have all kinds of reasons that do not fit the hassle that arXiv.org creates. I had two endorsers in 2009 but now had to find a new person in 2014, and then they arrogantly claim that they can judge my work. ArXiv.org disinform the world, as it does not provide what it says it does. Keeping a cross reference to EWS out since 2009 is a big shame, and mathematics education has been suffering the consequences. ArXiv.org is a shame. It uses taxpayer and fund money so that moderators can feel important and satisfy their religious fears for cranks as the modern devils and heretics.

Gibbs refers hesitantly to Philica as an outlet.⁴¹⁸ He also mentions that Timothy Gowers ⁴¹⁹ supports the French CCDS initiative to create 'epi-journals', i.e. journals that harvest from the archives,⁴²⁰ but he also mentions that this has a dark side.⁴²¹ CCDS restricts use to arXiv.org and French HAL, so they will not be able to use my work that is abused by arXiv.org.

Thus, my suggestion is to use *PressForward – the WordPress Plugin* – that would have the flexibility that CCDS doesn't provide. See also this discussion.⁴²²

PM 1. Gibbs rightly mentions that chess is not a universal result in mathematics.⁴²³ However, Elo-ranking in chess is a universal method to combine both competence and

⁴¹⁴ <http://arxiv.org/abs/0807.2656>

⁴¹⁵ <http://arxiv.org/abs/1408.1930>

⁴¹⁶ <http://vixra.org/info>

⁴¹⁷ <http://blog.vixra.org/2010/07/11/they-call-me-a-crank-now-lol/>

⁴¹⁸ <http://www.philica.com/>

⁴¹⁹ <http://gowers.wordpress.com/2013/01/16/why-ive-also-joined-the-good-guys/>

⁴²⁰ <http://episciences.org/>

⁴²¹ <http://blog.vixra.org/2013/01/18/the-darker-side-of-open-access/>

⁴²² <http://pressforward.org/discovering-scholarship/>

challenge. When we turn peer review into a match between author and reviewer,⁴²⁴ then Elo-ranks for that field of enquiry might gain some stability. See *Voting Theory for Democracy* chapter 7 on Elo-ranking or the identical Rasch (Item Response) model (but no details on the review match idea). See here⁴²⁵ for my chess match with Vladimir Putin.

PM 2. Gibbs also mentions that mathematical results should be less controversial, since it is hard to argue with a proof. He mentions issues on Cantor and Gödel as exception. Perhaps he already thinks that MER is excepted too ? On Cantor see my argument in favour of Occam,⁴²⁶ and on Gödel see my book *A Logic of Exceptions*. [Gibbs included a category on education.]

PM 3. See Gibbs on love and math,⁴²⁷ and consider why math teachers tend to kill the love for mathematics: they are not aware that education is an empirical issue and that it requires empirical research rather than merely a math degree.

PM 4. Raymond Johnson⁴²⁸ – a PhD student and former math teacher of *South Park High School* in Colorado – has this comment:

“Robert’s last recommendation is to have a preprint server for math education research. As he notes, this is a road we’ve tried to go down before and we didn’t get very far. I don’t think the problem has nearly as much to do with policy or categories of the arXiv as it does with the lack of a “preprint culture” in mathematics education. What I learned in those previous preprint discussions, and in my observations as a developing scholar, is that math educators regularly and happily share work in progress — *with a select group of people*. In math ed, there doesn’t seem to be widespread faith in anything like Linus’ Law,⁴²⁹ the open source software dictum that says, “With enough eyeballs, all bugs are shallow.” I think the math wars led to a lot of distrust, and some of it is very rational. It’s safer to only share preliminary work with a few scholars who share similar methods and theoretical frameworks, and then refine the work after peer review before publication in a journal whose readership is likely to understand the work. Maybe it shouldn’t be this way, but to move forward we’re going to have to confront some of these beliefs.” (RJ August 2014)

A problem here is that RJ might see too much 'math war' where there isn't one. He catalogs my criticism as such in his August 14 2014 blogtext.⁴³⁰ However my text is decent scientific criticism as you can verify here⁴³¹ – and also check *Elegance with Substance*. It is unscientific that RJ did not made a correction in his blogtext after my email to him asking for a correction. He is a supporter of the 'realistic mathematics education' approach, and one should expect that he looks into the discovery that it is a fraud.⁴³² However, the fiction by Hans Freudenthal is to him like the words of Jesus for that other author.

It appears very difficult for abstract thinking mathematicians to face reality, especially those adhering to the unscientific cult of 'realistic mathematics education'.

(Poster: "Uncle Hans (Freudenthal) is watching you". Poster distributed by devotees, Source: <https://storify.com/DrBote/math-methods-weekly>.)

⁴²³ <http://blog.vixra.org/2014/03/27/abel-prize-for-yokov-sinai/>

⁴²⁴ <http://publishing.mathforge.org/discussion/62/peer-review-cannot-fulfill-its-promise/>

⁴²⁵ <https://boycottholland.wordpress.com/2014/05/20/a-game-of-chess-at-the-kremlin/>

⁴²⁶ <http://thomascool.eu/Papers/ALOE/2012-03-26-CCPO-PCWA.pdf>

⁴²⁷ <http://blog.vixra.org/2013/10/20/book-review-love-and-math/>

⁴²⁸ <http://blog.mathed.net/2014/08/on-major-problems-and-grand-challenges.html>

⁴²⁹ http://en.wikipedia.org/wiki/Linus%27s_Law

⁴³⁰ <http://links.mathed.net/2014/08/freudenthals-realistic-mathematics.html>

⁴³¹ <http://arxiv.org/abs/1408.1930>

⁴³² <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

Dynamic logic for arXiv and viXra

2014-10-10 ⁴³³

In static logic p & q is equivalent to q & p . Dynamic logic arises when p && q differs from q && p . Perhaps “ordinal logic” is a term too. In programming, a list of statement $\{p[1], p[2], \dots, p[n]\}$ often is such that it matters in which order the statements are executed.

An example of execution indeed: first feed the prisoner and then hang him, or first hang him and then feed him: it tends to make a difference (except when the hanging fails and the prisoner survives, though possibly still with less of an appetite).

The wikipedia article on dynamic logic ⁴³⁴ distinguishes it from temporal logic but I have always used the term dynamic logic in that ordinal sense, see my book *A Logic of Exceptions* (ALOE), programmed in *Mathematica*.

Note also that the math students of MIT are sabotaging ⁴³⁵ the purposes of wikipedia more and more. While I wrote ALOE to arrive at a didactic flow, the wikipedia articles nowadays even on elementary issues plunge you into gibberish that is useless even for a decent math course. One wonders when it would dawn on the creators of wikipedia that there are Van Hiele levels of understanding, so that one would require at least three wikipedia articles: novice, student, professional.

Application of dynamic logic to yesterday's issue of arXiv versus viXra ⁴³⁶ leads to the following. An awful lot of research professionals attach value to arXiv rather than viXra & *PressForward*. These professionals tend to be decent scientists and are focused on dissemination. On occasion they might be worried about the odium of cranks. Thus, a sensible approach is:

- first use arXiv, and use viXra as a back-up if arXiv shows itself to be unscientific
- in viXra: use the comments to explain the history and invite reviews by others
- in viXra: for each submission also review at least one other work in your area
- use *PressForward* and Elo-rating (see *Voting Theory for Democracy*, chapter 7)

Phil Gibbs gave the examples of Cantor and Gödel. Both ended as nutcases but many will regard them as decent scientists in their published work. Some of their results are even highly regarded. When I looked at those claimed major elementary results, I found those wanting. On occasion I will regard Cantor and Gödel as crackpots of the first order, since if they had had common sense then they surely had the capacity to debunk their claimed 'big results'. I suppose that it may be merely human to delude oneself, and continue in that state if nobody corrects you. Vladimir Putin is a case in point. I also think that Kronecker failed as a scientist, since he should have given the counterargument instead of vilifying Cantor at a personal level. Somewhat differently, Hilbert embraced both Cantor and Gödel, and this shows that mathematics and common sense are different issues. Still, all those 'results' in arXiv that build on these 'big results' may to some extent be reclassified as pretty cranky. My proposal is that arXiv indeed tries for that reclassification, so that in the end arXiv and viXra may indeed not look all too different.

The issue can be classified in the following table. While Jesus is definitely not science, even though viXra currently allows for him, some people would still attach sense to him. At moments, novelists claim that their stories and novels attain a higher truth. Perhaps it is only an issue of the manner of communication and the speaking of a different

⁴³³ <https://boycottholland.wordpress.com/2014/10/10/dynamic-logic-for-arxiv-and-vixra/>

⁴³⁴ http://en.wikipedia.org/wiki/Dynamic_logic_%28modal_logic%29

⁴³⁵ <https://boycottholland.wordpress.com/2013/02/18/wikipedia-acrobatics/>

⁴³⁶ <https://boycottholland.wordpress.com/2014/10/09/replace-arxiv-by-vixra-and-pressforward/>

language. Earlier we discussed how 'historian' Amir Alexander first selects a good story and then collects the facts to support it.⁴³⁷ Such claims can always be debunked and that would be good science again (within brackets).

	Science	Not science
Sense	My work in economics, some elementary statistics and the education of mathematics. See the refutations in the links in the other boxes. Of course, my paper at viXra. ⁴³⁸	Jesus (SMOJ) ⁴³⁹ ?
Not so sensible or sheer nonsense (debunked)	<ul style="list-style-type: none"> ♦ Gödel with his Gödeliar (ALOE) ♦ Cantor's diagonal ($N < R$) (FMNAI) ♦ Cauchy & Weierstraß derivative (COTP) ♦ Arrow's impossibility theorem (VTFD) ♦ Tall's reinvention of Van Hiele (⁴⁴⁰) ♦ The Economist 100 years Einstein (in COTP) 	<ul style="list-style-type: none"> ♦ Hans Freudenthal's fraud (⁴⁴¹) ♦ cranks (that should put the fear of God into you)

⁴³⁷ <https://boycottholland.wordpress.com/2014/06/14/amir-alexander-and-history-as-storytelling/>

⁴³⁸ <http://vixra.org/abs/1410.0046> – now superseded by the book FMNAI

⁴³⁹ <http://thomascool.eu/Papers/SMOJ/Index.html>

⁴⁴⁰ <http://arxiv.org/abs/1408.1930>

⁴⁴¹ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

Pronunciation of numbers

Mathematical pronunciation of numbers in English, German, French, Dutch and Danish

2015-08-29 ⁴⁴²

An earlier weblog discussed that *English is a dialect of mathematics*. ⁴⁴³ Compare:

Number	Math	English
14	ten·four	fourteen
21	two·ten·one	twenty-one

Professor Fred Schuh ⁴⁴⁴ of TU Delft wrote about the different mathematical pronunciation in books in 1943 and 1949, and addressed the Dutch minister of education in 1952 (see here ⁴⁴⁵). Many others observed this issue too. There is a gap though between understanding the idea given by these few examples and seeing it developed fully. Thus I decided to write out the alternative.

The issue is discussed in *A child wants nice and no mean numbers*. ⁴⁴⁶ For some particular languages, the suggestions are in *Marcus learns to count with ten*. Select your language (text in English, numbers in the particular language):

English ⁴⁴⁷ – *German* ⁴⁴⁸ – *French* ⁴⁴⁹ – *Dutch* ⁴⁵⁰ – *Danish* ⁴⁵¹

The devil hides in the details

When you look at details then subtle problems show up. First of all, it appears better to use the middle dot instead of the hyphen, to prevent confusion with the negative sign.

Secondly, when mathematical pronunciation in German would use *zehn* for 10, then 90 in math would be *neun·zehn*, which would conflict with the current use of *neunzehn* for 19. You cannot propose such a change because it would create confusion. Germans would have to ask each other continuously: “Are you speaking math or dialect?”

There is already a regularity in German for the numbers of ten 20, 30, 40, ..., 90: *zwanzig*, ..., *neunzig*. Hence, German has at least these options: (a) adopt English *ten* for 10, (b) use *zig* for 10. The latter would give least change.

Number	Math in English	English	Math in German ?	German	Math in German !
19	ten·nine	nineteen	zehn·neun	neunzehn	zig·neun
90	nine·ten	ninety	neun·zehn	neunzig	neun·zig

A compromise would be to accept 10 = *zehn* = *zig*, and to use *zehn* up to 20 and *zig* from 20 onwards. When you are accepting change then rather do it properly though. My suggestion is to use *zig*. Dutch has the same problem, and here my suggestion is to use *tig*.

⁴⁴² <https://boycottholland.wordpress.com/2015/08/29/mathematical-pronunciation-of-numbers/>

⁴⁴³ <https://boycottholland.wordpress.com/2012/04/01/english-as-a-dialect-of-mathematics/>

⁴⁴⁴ https://en.wikipedia.org/wiki/Frederik_Schuh

⁴⁴⁵ <http://www.t-en-r.nl/trwww-archief/tr-archief-wiskunde/Schuh.htm>

⁴⁴⁶ <http://thomascool.eu/Papers/NiceNumbers/Index.html>

⁴⁴⁷ <http://thomascool.eu/Papers/NiceNumbers/2015-08-20-CWNN.pdf>

⁴⁴⁸ <http://thomascool.eu/Papers/NiceNumbers/2015-08-28-Marcus-in-Deutsch.pdf>

⁴⁴⁹ <http://thomascool.eu/Papers/NiceNumbers/2015-08-28-Marcus-en-Francais.pdf>

⁴⁵⁰ <http://thomascool.eu/Papers/Math/2012-03-29-EnzovoortsMarcus.pdf>

⁴⁵¹ <http://thomascool.eu/Papers/NiceNumbers/2015-08-28-Marcus-in-Danish.pdf>

Danish might use their current word *ti* for 10. However, I have listened in Google Translate⁴⁵² for the Danish pronunciation of *ti-ti* for 100, and though it sounds like *tea-tea*, I find it less convincing. My proposal for Danish is to use *ten*, which they already use for the numbers 13-19.

English actually has the same issue. The numbers of ten 20-90 use *ty* (e.g. forty, fifty), so that we might consider using *ty* instead of *ten*. This would give least change as in German. Then 90 would be *nine-ty* instead of *nine-ten*. However, English *ty-ty* for 100 is less convincing again. Thus *ten* for English is best.

German, Dutch and Danish might all adopt English *ten*. They already adopt *Google* or *computer*, and it would be curious when they are prim on 10, while change would benefit the learning of arithmetic by their young children enormously.

French can use *dix* for 10 without problem. French has some curious twists and turns. It suddenly relies on addition (*soixante-dix* = 60 + 10 for *sept-dix* = 7 × 10) and then switches to multiplication with 20 (80 = *quatre-vingts* for *huit-dix* = 8 × 10). When 20 changes from *vingt* to *deux-dix*, then it becomes advisable to change the whole system.

Accepting responsibility

Overall, we again see that mathematicians are trained for abstract thought and have insufficient awareness of the empirical realities of education. Mathematicians should explain to both teachers and *language managers* about the difference between mathematical pronunciation and national dialects. The problem doesn't necessarily lie in education but rather in mathematical neglect.

Professor Fred Schuh explained much of this already in 1943-1952, and thus one can argue that mathematics did explain it to education, so that it is the responsibility of education that they did not make the change. This is too simple a view.

This simple view does not square with devoted teachers who explain to their beloved pupils that *soixante-quinze* + *seize* = *quatre-vingt et onze* (check the confusing hyphen), in the belief that they are doing perfect arithmetic. These teachers should have had proper mathematics education, so that they know that they are short-changing their pupils.

Mathematics education should accept its responsibility and free itself from the stranglehold by the abstract thinking mathematicians who have no idea about the empirics of education. (See my earlier letter to IMU / ICMI.⁴⁵³)

(picture)

Fred Schuh painted by Han van Meegeren⁴⁵⁴ in the Dutch hunger period during World War II (source: Schuh's book of 1949: "De macht van het getal" ("The power of numbers"))

⁴⁵² <https://translate.google.com/#da/en/ti%20ti>

⁴⁵³ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

⁴⁵⁴ https://en.wikipedia.org/wiki/Han_van_Meegeren

Research on number sense tends to be invalid

2015-08-30 ⁴⁵⁵

The preceding weblog text considered the pronunciation of numbers in English, German, French, Dutch and Danish. ⁴⁵⁶

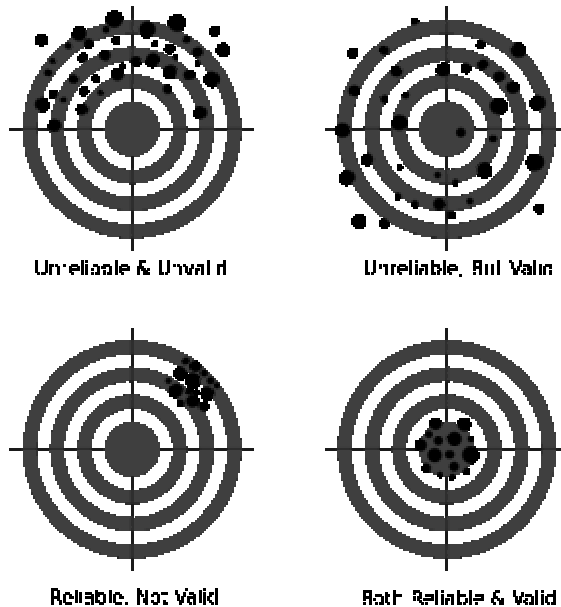
There better be a general warning about invalidity of current research on number sense.

Warning 1. The object of study concerns a chaotic situation

Research on how children learn numbers, counting and arithmetic, is mostly done in the context of the current confusing pronunciations. This is like studying people walking a tightrope while saying the alphabet in reverse order. This will not allow conclusions on the separate abilities: (a) dealing with arithmetic, (b) dealing with a confusing dialect.

In methodological terms: common studies suffer from invalidity. (Wikipedia. ⁴⁵⁷) They aren't targeted at their research objective: number sense. Perhaps they intend to, but they are shooting into a fog, and they cannot be on target.

A positive exception is this article by Lisser Rye Ejersbo and Morten Misfeldt (2015), "*The relationship between number names and number concepts*". ⁴⁵⁸ They provide pupils with the mathematical names of numbers and study how this improves their competence. This reduces the chaos that other studies leave intact.



Validity and reliability (source: wikimedia commons)

⁴⁵⁵ <https://boycottholland.wordpress.com/2015/08/29/research-on-number-sense-tends-to-be-invalid/>

⁴⁵⁶ <https://boycottholland.wordpress.com/2015/08/29/mathematical-pronunciation-of-numbers/>

⁴⁵⁷ https://en.wikipedia.org/wiki/Validity_%28statistics%29

⁴⁵⁸ <http://pure.au.dk/portal/en/persons/lisser-rye-ejersbo%28b66e2df6-0692-4c95-9d99-10fb812a4cf5%29/publications/the-relationship-between-number-names-and-number-concepts%28029859a5-697b-4d92-8750-62435f58075a%29.html>

It is insufficient to state that you want to study “*number sense in the current situation*”. When you grow aware that the current situation seriously hinders number sense, then you ought to see that your research objective is invalid, since the current situation confuses number sense. If you still want to study number sense in the current situation, hit yourself with a hammer, since apparently this is the only thing that will still stop you.

Warning 2. Results will be useless

Results of studies within the current chaos will tend to be useless: (a) They cannot be used w.r.t. mathematical pronunciation, since they don't study this. (b) Once the mathematical pronunciation is implemented, results on number sense within the current chaotic situation are irrelevant.

Warning Sub 2. Don't be confused by a possible exception

There seems to be one exception to warning 2: the comparison of English, which has low chaos in pronunciation, to other situations with higher chaos (Dutch, German, French, Danish). This presumes similar setup of studies, and would only be able to show that mathematical pronunciation indeed is better. Which we already know. It is like establishing over and over again that drinking affects driving. The usefulness of this kind of study thus must be doubted too. One should not be confused in thinking that it would be useful.

Indeed, we might imagine a diagram with a horizontal axis giving skill in addition with outcomes in the range 10-20 and a vertical axis giving skill in addition with outcomes in the range 20-50, both giving the ages when satisfactory skills have been attained, and then plot the results for English, German, French, Dutch and Danish. We would see that English has lower ages, and French might actually do better than German, since the strange French number names are for 70-99. It might make for a nice diagram, but the specific locations don't really matter since we already know the main message.⁴⁵⁹

For example, Xenidou-Dervou (2015:14) states:

“Increasingly more studies are suggesting that this inconsistency between spoken and written numbers can have negative effects on school-aged children's symbolic processing (e.g., Helmreich et al., 2011).”

Compare this with our earlier observation⁴⁶⁰ that professor Fred Schuh of TU Delft already proposed on these grounds a reform of pronunciation in Dutch in 1943, 1949 and 1952 ... Parliament in Norway (their “Storting”) decided in July 1950 to rename the numbers above 20 in English fashion.

It is not only problematic that Xenidou-Dervou isn't aware of this, but also that she doesn't see that the current chaotic situation invalidates her own research setup.

She remarks (2015:14) that the logical clarity (Schuh's insight) has not been subjected to statistical testing. This may be true. When you don't understand that drinking affects driving, then you might require statistics. Doing such tests is as relevant as statistical research on verifying that drinking affects driving. She states (my emphasis):

“To the best of our knowledge, the effect that the language of numbers can have in the development of a core system of numerical cognition such as children's symbolic approximation skills [using Arabic numbers], *controlling for their*

⁴⁵⁹ Ann Dowker et al. recently compared English and Welsh, <http://www.psy.ox.ac.uk/publications/535741>, and English and Cantonese, <http://www.psy.ox.ac.uk/publications/514955>, which other languages have number names that better conform to the mathematical pronunciation. I read the summaries with interest but also observed that they mainly confirm what logic already requires. A quick fix for education is to use Cantonese.

⁴⁶⁰ <https://boycottholland.wordpress.com/2015/08/29/mathematical-pronunciation-of-numbers/>

nonsymbolic approximation skills [using representations like dots but apparently not fingers] has not been previously addressed.”

Thus, the statistics on drunk driving are corrected for the performance on drunk riding a bicycle. It might be suggested that nonsymbolic number sense would be independent from language, and we might readily accept this for numbers smaller than 10, but to properly test this for 11-99 we need a large sample of *Kaspar Hausers*⁴⁶¹ who are unaffected by language. Xenidou-Dervou's correction does not remove the contamination by language.

Statistical tests may indeed be used to establish that large males tend to have a higher tolerance for drinking than small females, and to test legal standards. But questions like these are not at issue in the topic of number sense.

The relevant points are:

- It is already logically obvious that a change to mathematical pronunciation will be beneficial. There is no need for statistical confirmation, e.g. by comparing English with other language situations. To suggest that such research would be necessary is distractive w.r.t. the real scientific question (see next).
- The study of number sense can only be done validly in a situation with mathematical pronunciation, without the noise of the current chaotic situation of the national language dialects.

(PM. This is inverse of the case that there was statistical information that smoking was highly correlated with lung cancer, but that the tobacco industry insisted upon biological evidence. This analogy might arise when researchers would have stacks of statistical results proving that weird pronunciation is highly correlated with slow acquisition of mathematical understanding and skill, while there would be a strong lobby for maintaining national pronunciation who insist upon biological evidence. Thus do not confuse these statistical situations.)

Curiously, the press-release⁴⁶² on Xenidou-Dervou's promotion event and publication of the thesis of January 7 2015 states that she 'discovered' something which was already well known to Fred Schuh in 1943, 1949, 1952, if not some present-day teachers and children themselves:

“From age 5 the influence of teaching is larger than of natural abilities. What hinders Dutch children is the way how numbers are pronounced in Dutch. These relations have been found by Iro Xenidou-Dervou (...)”

“One of the teachers in the researched schools could confirm this with an anecdote from practice. She had heard one pupil telling another pupil doing a calculation: “Do it in English, that is easier.””

“Xenidou-Dervou thus suggests to start in Holland with education in symbolic calculation [with Arabic numbers] already before First Grade [age 6].”

Perhaps we might already start with Arabic numbers before First Grade indeed. Some children already watch Sesame Street. It would be more advisable to do something about pronunciation however. It is perhaps difficult to maintain common sense when you are in a straight-jacket of thesis research.

⁴⁶¹ https://en.wikipedia.org/wiki/Kaspar_Hauser

⁴⁶² <http://www.vu.nl/nl/nieuws-agenda/nieuws/2015/jan-mrt/rekenonderwijs-meest-bepalend-voor-rekenprestaties.asp>

Warning 3. Such studies will not discover the true cause for the current chaotic situation

The barrier against the use of mathematical pronunciation doesn't lie with the competences of children but with the national decision making structure. Thus, most current studies on education and number sense will never discover, let alone resolve, the true problem.

That the mathematical pronunciation will be advantageous is crystal clear. Of course it helps when you are allowed to first walk the tightrope and only then say the alphabet in reverse. Thus we have to look at the national decision making structure to see why this isn't done.

Of key importance are misconceptions about mathematicians. Policy makers and education researchers often think that mathematicians know what they are doing while they don't. Education researchers may be psychologists with limited interest in mathematics per se. Few are critical of what children actually must learn.

We may accept that psychology is something else than mathematics education, but when a psychologist researches the education of mathematics then we ought to presume that they know about mathematics education. When they don't understand mathematics education then they should not try to force it into their psychological mold, and go study something else.

Two relevant books of mine on this issue are:

- *A child wants nice and not mean numbers*,⁴⁶³ for primary education
- *Elegance with Substance*,⁴⁶⁴ for education in general but targeted at highschool and first year of tertiary education.

Warning 4. Mathematics education research has breaches of scientific integrity

Current research on education and number sense assumes that there is an environment with integrity of science. However, there is a serious breach by Hans Freudenthal (1905-1990) w.r.t. the results of his Ph. D. student Pierre van Hiele (1909-2010).⁴⁶⁵ Van Hiele discovered the key educational relevance of the distinction between concrete versus abstract, with levels of insight, while Freudenthal interpreted that as the distinction between applied and pure mathematics, and henceforth used his elbows to get Van Hiele out of the way. Freudenthal was an abstract thinking mathematician who invented his own reality. There now exists a Freudenthal "Head in the Clouds Realistic Mathematics" Institute in Utrecht. Its employees behave as a sect, reject criticism, will not look into Freudenthal's breach of integrity of science, and will not undo the damage. See my letter to IMU / ICMI.⁴⁶⁶ Other researchers tend not to know about this, and tend to accept "findings" from Utrecht assuming that it has a "good reputation".

This warning holds in general

Just to be sure: this warning on invalidity of research on number sense is general. We might for example think of issues discussed in the *Oxford Handbook of Numerical Cognition* (2015), edited by Ann Dowker. Or think about issues discussed by Korbinian

⁴⁶³ <http://thomascool.eu/Papers/NiceNumbers/Index.html>

⁴⁶⁴ <http://thomascool.eu/Papers/Math/Index.html>

⁴⁶⁵ <https://boycottholland.wordpress.com/2014/07/06/hans-freudenthal-s-fraud/>

⁴⁶⁶ <https://boycottholland.wordpress.com/2014/09/02/for-imu-icmi-integrity-of-science-in-dutch-research-in-didactics-of-mathematics/>

Moeller et al. (2011),⁴⁶⁷ or Elise Klein et al. (2013).⁴⁶⁸ But, this weblog is about a major problem in Holland, and thus it might help to make some remarks concerning the anatomy of Holland.

Comment w.r.t. the Dutch MathChild project

The Dutch MathChild project can be found here,⁴⁶⁹ with contacts in Belgium, UK and Canada. Its background is in psychology and not in mathematics education.

The Amsterdam thesis by Iro Xenidou-Dervou (2015) isn't fully online and it should be.⁴⁷⁰

There is the full thesis by Ilona Friso-Van den Bos (2014).⁴⁷¹ She did the thesis at the dept. of education & pedagogy in Utrecht, but now she is at the Freudenthal "Head in the Clouds Realistic Mathematics" Institute (FHCRI). I looked at this thesis only diagonally. Issues quickly become technical and this is secondary to the first question about validity. At first glance the thesis does not show sect behaviour (allowing for contagion from FHCRI to other places at Utrecht University). The names of Freudenthal and Van Hiele are not in the thesis. The thesis has a neuro-psychological setup with a focus on working memory, which suggests some distance from mathematics education. The scheme of the thesis is that you define a test for number sense, a test for working memory, and a test for mathematical proficiency (try to imagine this without number sense and working memory), and then use children to see what model parameters can be estimated. Criticism 1 is that "mathematics achievement" is in the title and used frequently (see also the picture on p282), and taken for Holland as the CITO score (p160), which has a high FHCRI content (so we find contagion indeed). Criticism 2 is that working memory belongs to the current fashion in neuro-psychology but is less relevant for mathematics education. For ME it is important to get rid of Freudenthal's misconceptions and to look at Van Hiele levels of insight. Thus, get proper use of working memory, rather than train it to become a bit larger to do crummy FHCRI math.

Criticism 3 concerns our present issue: the handling of the pronunciation of numbers. The thesis gives:

"(...) a difference between participants from linguistic backgrounds in which number words are inverted (e.g., saying six-and-twenty instead of twenty-six), because these inversions have been suggested to be a source of difficulty in number processing (Klein et al., 2013), and that errors related to inversion can be associated with central executive performance (Zuber, Pixner, Moeller, & Nuerk, 2009)." (p82)

"Publication year and inversion of number words did not play a role in the prediction of effect sizes." (p97)

On p197-198 we find, my emphasis:

"An alternative explanation for the deviation in findings between previous studies (e.g., Barth & Paladino, 2011) and the current study is that in all previous studies, children were taught in English, in which the number system is more uniform than the Dutch number system. Dutch number words include the ones before the tens, instead of tens before ones (e.g., instead of saying thirty-five, one would say five-and-thirty), which is inconsistent with the order of written numerals. **This may make it more difficult for young children to gain insight**

⁴⁶⁷ <http://www.ncbi.nlm.nih.gov/pubmed/21498043>

⁴⁶⁸ <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3733006/>

⁴⁶⁹ <http://vu.mathchild.nl/en/home/>

⁴⁷⁰ <http://dare.ubvu.vu.nl/handle/1871/52176>

⁴⁷¹ [http://www.narcis.nl/publication/RecordID/oai%3Adspace.library.uu.nl%3A1874%2F297856/id/1/Language/NL/uquery/Making sense of numbers/coll/publication](http://www.narcis.nl/publication/RecordID/oai%3Adspace.library.uu.nl%3A1874%2F297856/id/1/Language/NL/uquery/Making%20sense%20of%20numbers/coll/publication)

into the number system, and might explain the large number of children being placed in the random group during kindergarten, leading children to prevail in using less mature placement strategies and skipping the strategy with three reference points to inform number line placements in favour of the most advanced strategy, which is making linear placements. This hypothesis, however, rests under the assumption that children make placements through interpretation of verbal number words, either by transcoding the written number or by listening closely to the experimenter reading the numbers out loud. A study by Helmreich et al. (2011) indeed suggested that inversion errors may be of influence on number line placements in primary school children, although an important difference with the current study was that no numbers were read out loud by the experimenter, making the chance of inversion errors larger. **More experimental studies are needed** to investigate similar differences in findings and manipulate strategy use through variations in instruction in various groups.”

Criticism 3 thus generates the sub-criticisms:

1. It is not only problematic that Friso-Van den Bos doesn't give the earlier reference to professor Fred Schuh of TU Delft in 1943, 1949 and 1952, but also that she doesn't see that the current chaotic situation invalidates her own research setup. Yes, we do see that she makes a correction at times, but the point is that the proper correction is that the thesis as a whole is shelved, since the situation that she studies cannot render the data that she needs.
2. It is curious that she states that “more experimental studies are needed”. Compare this with a study of drunken driving in London, Paris, Oslo, Athens, ... to test whether there are differences ... I cannot understand how an educator can observe the crooked pronunciation of numbers, and not see immediately how important it is to *remove* the bottleneck rather than further *research* it. This is like finding a cancer and not remove it but argue that it needs more study. One might say that it is “only a Ph. D. study”, but the idea of a dissertation is that it shows that one can do scientific research by oneself individually. A researcher should be able to spot issues on validity. (Perhaps most Ph. D. students are too young or perhaps standards are too low given current academic culture.)

Concluding on the responsibility of educators of mathematics

As in the earlier weblog text, the main responsibility lies with Parliament: to investigate the issue.

It will still be the educators of mathematics who have the responsibility to re-engineer the mathematical pronunciation of numbers, to be used in education, and subsequently also in society and courts of justice. As a teacher of mathematics, I have presented my suggestions in the earlier weblog text, see here.⁴⁷²

⁴⁷² <https://boycottholland.wordpress.com/2015/08/29/mathematical-pronunciation-of-numbers/>

Part 4.

Another reason to boycott Holland

Rules of engagement

This book is critical about breaches of scientific integrity in Holland in the field of mathematics and mathematics education research (MMER).

Hence, this book is at risk of being *overly critical* and of being *needlessly negative* about all the good people who are doing their best to advance mathematics and improve mathematics education. Perhaps this book might have some useful points that deserve attention: but when the case is overstated then the book is counterproductive as well.

Hence it is useful to keep in mind some rules of engagement. The umbrella is given by science itself, with rules that would be obvious to any scientist. Those rules grow on you when you are doing research, and may be hard to recall explicitly. It may be useful to refer to some instances where others have tried to be more explicit.

Some useful rules of engagement are: ⁴⁷³

- (1) Back your argument up
- (2) Respect other people's work
- (3) Know thyself
- (4) Be open to other ideas
- (5) Stick to the point
- (6) Discuss

Some reflections of my own would be:

- (7) Treat people like you would want to be treated yourself (as in computer programming: do not program what you don't want to be programmed to yourself)
- (8) Do not generalize (without purpose and evidence). Be specific
- (9) Inform people about criticism and allow them a response
- (10) When you propose a change, then you have more information than others about the reasons for that change and about what it would entail: whence you have a special responsibility to make sure that you are clear on your proposal for a change
- (11) When you observe some generality – e.g. a phenomenon like "*mathematicians are trained on abstraction but when they enter a classroom then they meet with real life students*" – then treat this at the general level, for instance political economy and the need for a parliamentary enquiry, and still remain specific when you would have criticism w.r.t. a particular person. Even when the general property has been established at the general level, do not use this as an explanation at the personal level – e.g. X is abusive "because" (s)he is a mathematician – but formulate the criticism in a specific manner, and focus on behaviour. An example of fine criticism is: "Edward Frenkel in a presentation for a general audience (Speyer legacy school ⁴⁷⁴) perhaps uses the word "mathematics" in every other sentence as if this would explain anything."

There are the *Ethical Guidelines* of the *American Mathematical Society*: ⁴⁷⁵

"To assist in its chartered goal, "...the furtherance of the interests of mathematical scholarship and research ...", and to help in the preservation of that atmosphere of mutual trust and ethical behavior required for science to prosper, the Council of the American Mathematical Society sets forth the following ethical guidelines. These guidelines reflect its expectations of behavior both for AMS members, as well as for all individuals and institutions in the wider

⁴⁷³ Perhaps no longer available http://www.philosophical-investigations.org/The_Limits_of_Reason

⁴⁷⁴ <https://boycottholland.wordpress.com/2014/10/12/edward-frenkels-abuse-of-love-isnt-love/>

⁴⁷⁵ <http://www.ams.org/about-us/governance/policy-statements/sec-ethics>

mathematical community, including those engaged in the education or employment of mathematicians or in the publication of mathematics. These guidelines are not a complete expression of the principles that underlie them. The guidelines are not meant to be a complete list of all ethical issues. They will be modified and amplified by events and experience. These are guidelines, not a collection of rigid rules."

The area is a mine field of course. Once you start formulating guidelines and criteria, lawyers step in, and before you know there is litigation all over. Mathematics itself originated when lawyers looked at the rules for numbers and space. The combination of lawyers and mathematicians is ominous.

The US Dept. of Education formulated some draft rules on research integrity⁴⁷⁶ and got a response from Union of Concerned Scientists.⁴⁷⁷ Such pro's and con's may be more informative than rules that suggest that everything should be clear.

I am an econometrics researcher who got involved in mathematics education only later on. My tendency is to look first at the AMS, with research mathematicians (RM), and only later at the *National Council of Teachers of Mathematics* (NCTM).⁴⁷⁸ Observe that AMS is concerned about the training of mathematicians, whence they also published professor Wu's 2011 primer for elementary school.⁴⁷⁹ Somehow I also tend to look to Holland and the US and less at the EU. My focus on the AMS and its RM can be explained by the following two points, that are discussed in the paper *Neoclassical mathematics for the schools*, now included in FMNAI:

- Some of my results have consequences for mathematics itself, and I would be interested in what research mathematicians think of these. This concerns results on logic in ALOE, differential analysis with the dynamic quotient in ALOE / EWS / COTP, and an approach to infinity in ALOE / FMNAI. I explain the situation in the paper *What a mathematician might wish to know about my work*, now included in FMNAI. Perhaps teachers of mathematics (TM) might agree that those results are useful for math education, but when the RM protest that this would not be proper mathematics, then the TM will hesitate. It is difficult to teach something that is not proper mathematics even when it is didactically sound. This is a bottleneck for any innovation. (The proposals on trigonometry are less involved though.)
- There is the recent call in the AMS *Notices* by professor Wu (2011a) to RM to start participating actively in the education enterprise: but he calls for the wrong cavalry since we need *engineers* to help out. When RM have been trained on abstraction then they cannot deal with the empirical science of education research. It is important to get clarity of this at the AMS. Obviously, many RM also teach their subjects at the academia, and thus they will not be totally oblivious to some demands of empirics. Such issues are always balanced and moderate. But experience also shows that we should be wary about being unbalanced.

Incidentally, a search at NCTM.org on research integrity doesn't generate results, which might be a good sign. There is a suggestion of a professional oath for TM though.⁴⁸⁰

We find some guideliness also at National Public Radio (NPR), albeit for journalists:⁴⁸¹ accuracy, fairness, completeness, honesty, independence, impartiality, transparency, accountability, respect, excellence, putting principles into practice.

⁴⁷⁶ http://ies.ed.gov/pdf/EDScientificIntegrityPolicy_Draft.pdf

⁴⁷⁷ http://www.ucsusa.org/sites/default/files/legacy/assets/documents/scientific_integrity/ED-policy-analysis-comments.pdf

⁴⁷⁸ <http://www.nctm.org/>

⁴⁷⁹ <http://www.ams.org/publications/authors/books/postpub/mbk-79>

⁴⁸⁰ <http://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Johnny-Lott/A-Professional-Oath-for-Mathematics-Teachers/>

⁴⁸¹ <http://ethics.npr.org/>

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PM 1. Colignatus is the name of Thomas Cool in science. See <http://thomascool.eu>.

PM 2. "Legacy:source" means that an earlier reference has become superseded by a new edition in source.

PM 3. References in the footnotes and links need not be repeated here.

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